Rules for Exponents Logarithms and Radicals

Exponents

An exponent is a number that tells how many times the base is used as a factor of a term; in an expression of the form a^n, n is called the exponent, a is the base. In the expression a^2, 2 is the exponent and indicates that the base a is used as a factor twice $a \cdot a$.

1. Multiplication: $a^n a^m = a^{n+m}$
   Example: $a^2 a^3 = a^5$
2. Division: $\frac{a^n}{a^m} = a^{n-m}$
   Example: $\frac{a^5}{a^3} = a^2$
3. Power rule: $(a^n)^m = a^{nm}$
   Example: $(a^2)^3 = a^6$
4. Distributed over a simple product: $(ab)^n = a^n b^n$
   Example: $\left(ab\right)^2 = a^2 b^2$.
5. Distributed over a complex product: $(a^m a^p)^n = a^{mn} b^{pn}$
   Example: $(a^3 a^2)^2 = a^6 b^4$
6. Distributed over a simple quotient: $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
   Example: $\left(\frac{a}{b}\right)^2 = \frac{a^2}{b^2}$
7. Distributed over a complex quotient: $\left(\frac{a^m}{b^p}\right)^n = \frac{a^{mn}}{b^{pn}}$
8. Example: $\left(\frac{a^3}{b^4}\right)^2 = \frac{a^6}{b^8}$
9. Negative exponent: $a^{-n} = \frac{1}{a^n}, a^{-1} = \frac{1}{a}$
   Example: $a^{-2} = \frac{1}{a^2}$
10. Exponent of zero: $a^0 = 1$
    Example: $5^0 = 1$
11. No exponent: $a = a^1$
    Example: $5 = 5^1$

12. Fractional exponent: $a^{\frac{1}{n}} = \sqrt[n]{a}$

Example: $a^{\frac{1}{3}} = \sqrt[3]{a}$

13. Negative fractional exponent: $a^{-\frac{m}{n}} = \frac{1}{a^{\frac{m}{n}}}$

Example: $a^{-\frac{2}{3}} = \frac{1}{a^{\frac{2}{3}}} = \frac{1}{\sqrt[3]{a^2}}$

14. Fractional exponent numerator $\neq 1$: $a^{\frac{m}{n}} = \sqrt[n]{a^m}$ or $\left(\sqrt[n]{a}\right)^m$, $(a \geq 0)$

Example: $a^{\frac{2}{3}} = \sqrt[3]{a^2}$

Radicals

A radical is an expression used to indicate the root of a number. The components of a radical are as follows:

$\sqrt[n]{a^m}$ Where n is the index, $\sqrt{\ }$ is the radical sign, and a is the radicand.

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers then:

1. If n is even, then: $\sqrt[n]{a^n} = |a|$

   Example: $\sqrt[2]{5^2} = |5|$
2. In n is odd, then: $\sqrt[n]{a^n} = a$

   Example: $\sqrt[2]{5^2} = 5$
3. The product rule: $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$

   Example: $\sqrt[3]{a} \cdot \sqrt[3]{b} = \sqrt[3]{ab}$
4. The quotient rule: $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

   Example: $\sqrt[4]{\frac{a}{b}} = \frac{\sqrt[4]{a}}{\sqrt[4]{b}}$
5. The product rule with coefficients: $a\sqrt{b} \cdot c\sqrt{d} = ac\sqrt{bd}$

   Example: $2\sqrt{7} \cdot 3\sqrt{7} = 6\sqrt{49} = 6(7) = 42$
6. The sum of square roots (same index and radicand): $\sqrt{a} + \sqrt{a} = 2\sqrt{a}$

   Example: $\sqrt{8} + \sqrt{8} = 2\sqrt{8} = 2(2)\sqrt{2} = 4\sqrt{2}$
7. Nested radicals: $\sqrt[n]{\sqrt[m]{a}} = \sqrt[nm]{a}$

   Example: $\sqrt[2]{\sqrt[3]{a}} = \sqrt[6]{a}$
8. If $x^2 = p$ then $x = \pm\sqrt{p}$
9. Negative signs and exponents:

   $(-a)^2 = (-a) \cdot (-a) = a^2$ Example: $(-5)^2 = (-5) \cdot (-5) = 25$

   $-a^2 = -(a \cdot a) = -a^2$ Example: $-5^2 = -(5 \cdot 5) = -25$

Logarithms

A logarithm is the exponent, *n*, to which the base *b* must be raised to equal *a*, written as $\log_b a = n$.

Example: $\log_2 32 = 5$ This is read "the log of 32 to the base 2 is 5."

Rules of Logarithms

1. $\ln 1 = 0$, $\log_b 1 = 0$
2. $\ln e = 1$, $\log_b b = 1$
3. $\ln e^x = x$, $\log_b b^x = x$
4. If $b^x = y$ and $b > 0$ then $y = \log_b x$

   Example: $\log_5 125 = 3$ because $5^3 = 125$
5. Logarithm of a base to a power: $\log_b b^x = x$
6. Base to a logarithm: $b^{\log_b x} = x$
7. Notation for logarithm base 10: $\log x = \log_{10} x$
8. Notation for logarithm base *e*: $\ln x = \log_e x$
9. Product rule: $\log_b (MN) = \log_b M + \log_b N$

   Example: $\log_4 5x^3 y = \log_4 5 + \log_4 x^3 + \log_4 y$
10. Quotient rule: $\log_b \left(\frac{M}{N}\right) = \log_b M - \log_b N$

    Example: $\ln\left(\frac{\sqrt{3x-5}}{7}\right) = \ln(3x-5)^{\frac{1}{2}} - \ln 7$
11. Power rule: $\log_b N^p = p \log_b N$, $\ln y^x = x \ln y$

    Example: $\log_4 x^3 = 3 \log_4 x$
12. Change of base formula: $\log_b N = \frac{\log_a N}{\log_a b}$

    Example: $\log_4 25 = \frac{\log 25}{\log 4} \approx \frac{1.39794}{0.60206} \approx 2.3219$