Rules for Fractions

If $\frac{a}{b}$ and $\frac{c}{d}$ are fractions with $b \neq 0$ and $d \neq 0$ then:

1. Equality: $\frac{a}{b} = \frac{c}{d} \Leftrightarrow ad = bc$

   Example: $\frac{1}{2} = \frac{4}{8} = (1)(8) = (4)(2) = 8$

2. Equivalency: $\frac{a}{b} \cdot \frac{c}{c} = \frac{ac}{bc}$

   Example: $\frac{2}{3} \cdot \frac{2}{2} = \frac{4}{6}$

3. Addition and subtraction (like denominators): $\frac{a}{b} \pm \frac{c}{b} = \frac{a \pm c}{b}$

   Example: $\frac{2}{3} + \frac{3}{3} = \frac{5}{3}$

4. Addition and subtraction (unlike denominators): $\frac{a}{b} \pm \frac{c}{d} = \frac{ad}{bd} \pm \frac{cb}{bd} = \frac{ad \pm cb}{bd}$

   Example: $\frac{1}{2} + \frac{2}{3} = \frac{(1)(3)}{(2)(3)} + \frac{(2)(2)}{(2)(3)} = \frac{3+4}{6} = \frac{7}{6}$

5. Multiplication: $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$

   Example: $\frac{1}{2} \cdot \frac{2}{3} = \frac{2}{6} = \frac{1}{3}$

6. Division (invert and multiply): $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$

   Example: $\frac{1}{2} \div \frac{2}{3} = \frac{1}{2} \cdot \frac{3}{2} = \frac{3}{4}$

$$\frac{a}{\left(\frac{b}{c}\right)} = \frac{\left(\frac{a}{1}\right)}{\left(\frac{b}{c}\right)} = \left(\frac{a}{1}\right)\left(\frac{c}{b}\right) = \frac{ac}{b}$$

$$\frac{\left(\frac{a}{b}\right)}{c} = \frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{1}\right)} = \left(\frac{a}{b}\right)\left(\frac{1}{c}\right) = \frac{a}{bc}$$

7. Complex fractions:

$$\frac{\left(\frac{a}{b}\right)+\left(\frac{c}{d}\right)}{\left(\frac{e}{f}\right)+\left(\frac{g}{h}\right)}=\frac{\left(\frac{a}{b}\right)+\left(\frac{c}{d}\right)}{\left(\frac{e}{f}\right)+\left(\frac{g}{h}\right)}\cdot\frac{bdfh}{bdfh}=\frac{(ad+bc)\,fh}{(eh+fg)bd}$$

Example: $\dfrac{\left(\frac{1}{2}\right)+\left(\frac{2}{3}\right)}{\left(\frac{3}{4}\right)+\left(\frac{2}{5}\right)}=\dfrac{[(1)(3)+(2)(2)](4)(5)}{[(3)(5)+(4)(2)](2)(3)}=\dfrac{(3+4)20}{(15+8)6}=\dfrac{140}{138}=\dfrac{70}{69}$

Common Fraction Errors

$\dfrac{a}{b+c}\neq\dfrac{a}{b}+\dfrac{a}{c}$ But $\dfrac{b+c}{a}=\dfrac{b}{a}+\dfrac{c}{a}$

$\dfrac{a+bx}{a}\neq 1+bx$ The a in the numerator is not a factor

$\dfrac{a}{0}\neq 0$ Division by zero is undefined