Mathematics Dictionary

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A

AA triangle similarity (Angle/Angle) - If a one-to-one correspondence exists between the vertices of two triangles such that two angles of one triangle are congruent to the corresponding two angles of the second triangle, then the two triangles are similar

AAS triangle congruence (Angle/Angle/Side) - If a one-to-one correspondence exists between the vertices of two triangles such that two angles and the side opposite one of them in one triangle are congruent to the corresponding parts of the second triangle, then the two triangles are congruent

ASA triangle congruence (Angle/Side/Angle) - If a one-to-one correspondence exists between the vertices of two triangles such that two angles and the included side of one triangle are congruent to the corresponding parts of the second triangle then the two triangles are congruent

Abscissa - The horizontal or *x*-coordinate of a two-dimensional coordinate system

Absolute value - The distance from 0 to a number *n* on a number line; the absolute value of a number *n* is indicated by $|n|$

Example: $|-3| = 3$, $|+3| = 3$, and $|0| = 0$

Absolute value equation - An equation containing the absolute value of a variable

Example: $|x+3| = 9$

Absolute value function - A function containing the absolute function of a variable

Example: $f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

Absolute value inequality - An inequality containing the absolute value of a variable

Example: $|x+3| < 9$

Accuracy - How close a numerical measure is to its actual value; the quality of being near to the true value; the number of significant figures given in a number

Acute angle - An angle whose measure is greater than 0° and less than 90°

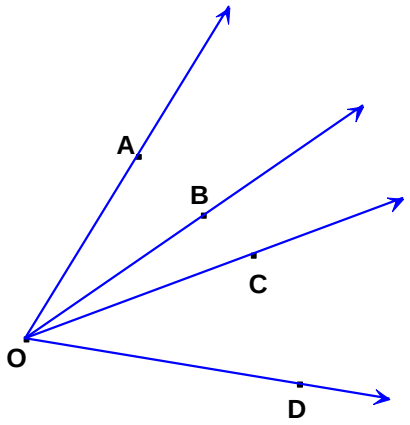
Acute triangle - A triangle that contains three acute angles; a triangle whose interior angles are all acute

Addition - to combine (numbers) into an equivalent simple quantity or number; a mathematical operation that represents combining collections of objects together into a larger collection; it is signified by the plus sign (+)

Additive property of equality - If a, b, and c are real numbers such that $a = b$, then $a + c = b + c$.

Adjacent angles - Two coplanar angles that share a common vertex and a common side but have no common interior points

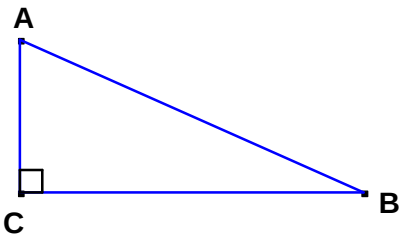
Example: In the figure below, $\angle AOB$ and $\angle BOC$ are a pair of adjacent angles, but $\angle AOC$ and $\angle BOD$ are not adjacent.



Adjacent sides – Any two sides of a polygon that share a common vertex

Adjacent side of an acute angle in a right triangle - The leg of the right triangle that is a side of the acute angle

Example: In right triangle *ABC* below, $\overline{AC}$ is adjacent to angle *A* and $\overline{BC}$ is adjacent to angle *B*



Algebra - The branch of mathematics that uses letters and/or symbols, to represent numbers and express mathematical relationships; the mathematics of generalized arithmetical operations

Algebraic equation - A mathematical statement that is written using one or more variables and constants which contains an equal sign.

Examples:

$3y + 5 = 1$

$\sqrt{2x - 5} = 11$

$\log_5(x - 3) = 2$

$2^x = \frac{1}{8}$

Algebraic expression - A mathematical phrase that is written using one or more variables and constants, but which does not contain a relation symbol ($<, >, \leq, \geq, =, \neq$)

Examples:

$3y+5$

$\sqrt{2x-5}$

$\log_5(x-3)$

2^x

Algebraic fraction - A fraction that contains an algebraic expression in its numerator and/or denominator.

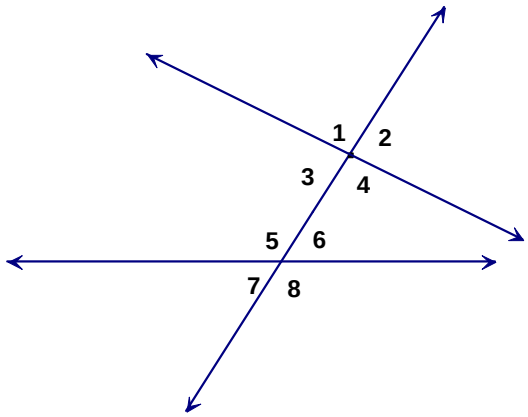
Example: $\dfrac{2x+4}{x+2}$

Algebraic representation - The use of an equation or algebraic expression to model a mathematical relationship

Algorithm - a defined series of steps for carrying out a computation or process; a precise rule (or set of rules) specifying how to solve some problem

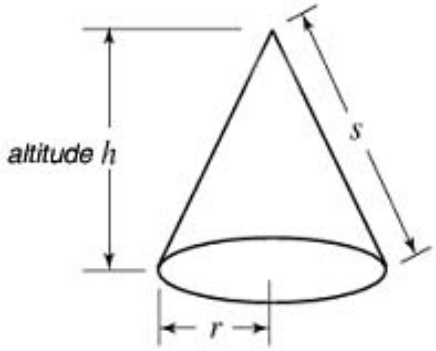
Alternate interior angles - Any two nonadjacent angles that lie on opposite sides of a transversal and that are interior to the lines. Note: As illustrated in the second example, the lines do *not* need to be parallel.

Example: In both diagrams below**,** $\angle 4$ and $\angle 5$, and $\angle 3$ and $\angle 6$ are pairs of alternate interior angles

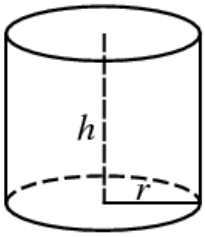


Altitude – the perpendicular distance from the base of a geometric figure to the opposite vertex (or side if parallel)

Of a cone: A line segment drawn from the vertex of the cone perpendicular to the plane containing its base:

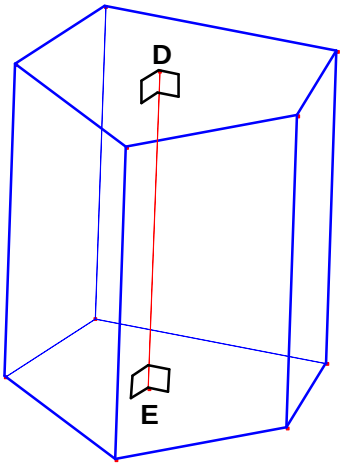


Of a cylinder: A line segment drawn from any point on one base of a cylinder perpendicular to the plane containing its other base.



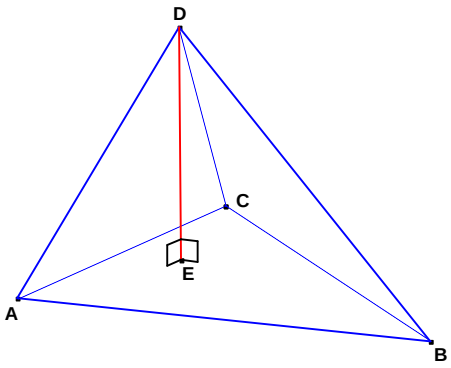
altitude h

Of a prism: A line segment drawn from any point of one base of the prism perpendicular to the plane containing its other base:



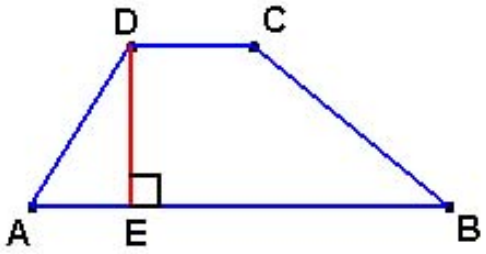
altitude $\overline{DE}$

Of a pyramid: A line segment drawn from the vertex of the pyramid perpendicular to the plane containing its base:



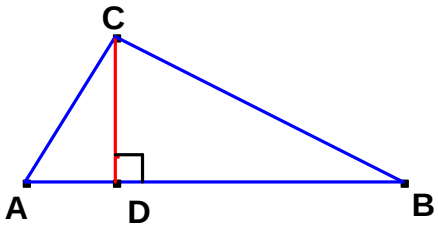
altitude $\overline{DE}$

Of a trapezoid: A line segment drawn from any point on one base of the trapezoid perpendicular to the other base:



altitude $\overline{DE}$

Of a triangle: A line segment drawn from any vertex of the triangle perpendicular to the line containing its opposite side:



altitude $\overline{CD}$

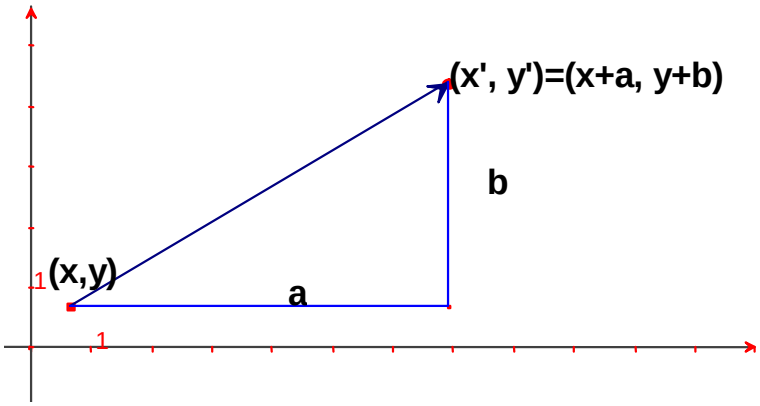
Ambiguous case - The case where the number of triangles found can vary from zero to two, when given two sides of a triangle and the measure of the angle opposite one of the sides

Amplitude - The magnitude of the oscillation of a sinusoidal function; the absolute value of one-half of the difference between the maximum and minimum function values of a sinusoidal function; the height of a wave

Analytical geometry - An approach to geometry in which the points of a figure are represented by coordinates on the Cartesian plane and algebraic methods of reasoning are used to study the figure

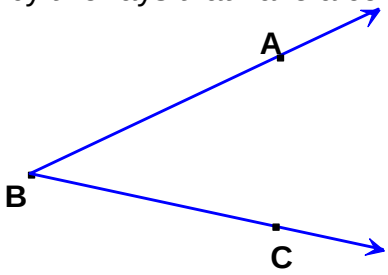
Analytical representation of a transformation - The functional notation of a transformation using analytical equations

Example: $T_{a,b}(x,y)=(x+a),(y+b)=(x',y')$ where $x'=x+a$ and $y'=y+b$ is a translation that moves points a units in the x direction and b units in the y direction. See figure below.



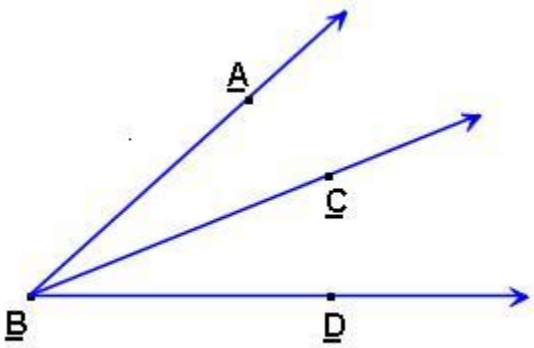
Analyze - to examine methodically by separating into parts and studying their relationships

Angle - A geometric figure formed by two rays that have a common endpoint



$\angle ABC$

Angle addition postulate - If $\angle ABC$ and $\angle CBD$ are adjacent angles then $\angle ABD=\angle ABC+\angle CBD$



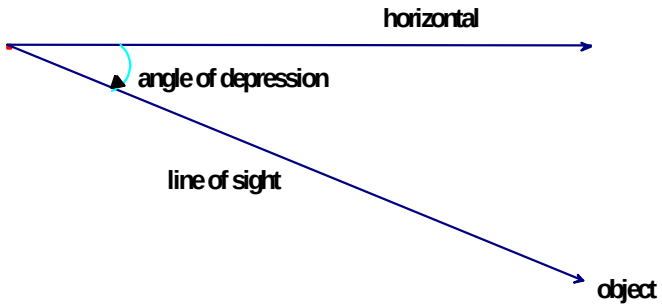
Example: $\angle ABD=\angle ABC+\angle CBD$

Angle bisector - A ray that divides an angle into two adjacent congruent angles

Angle in standard position - An angle with a vertex at the origin; the initial ray is on the positive x-axis, and the terminal side falls in one of the four quadrants or on one of the axes

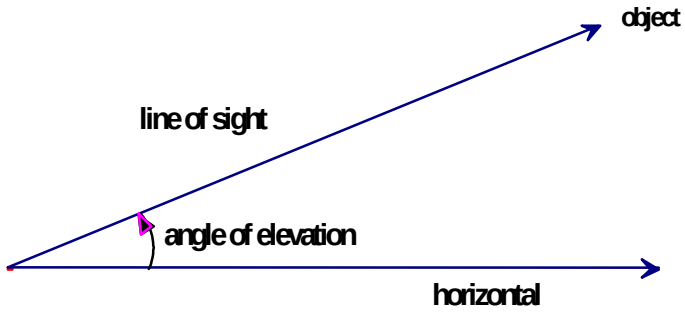
Angle measure - The number of degrees or radians in an angle

Angle of depression - The angle formed by the horizontal and the line of sight when looking downward



Angle of elevation - The angle formed by the horizontal and the line of sight when looking upward

Example:



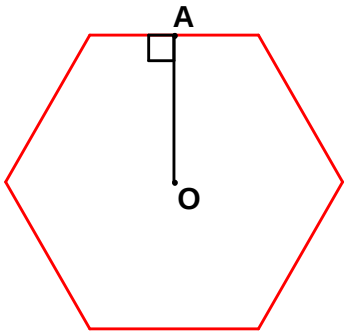
Antilogarithm - The inverse function of a logarithm; to find a number given its logarithm

Example: If $\log_{10}100 = 2$ then, 2 is the antilogarithm of 100

Antecedent - The "if" part of a conditional (if…, then…) statement

Apothem - A line segment drawn from the center of a regular polygon perpendicular to a side of the polygon

Example: apothem $\overline{OA}$



Approximate value - A value for some quantity, accurate to a specified degree

> **Example:** A board that measures 4 feet 2 inches has an approximate length to the nearest foot of 4 feet

Arc length - The distance on the circumference of a circle from one endpoint of an arc to the other endpoint, measured along the arc. For a circle of radius *r*, a central angle intercepts an arc of length *s* given by $s = r\theta$ where θ is measured in radians.

Arc measure -The measure of an arc of the circle in degrees or radians; a unique real number between 0 degrees and 360 degrees or between zero and 2π radians

Arc of a circle - See major arc, minor arc

Arccosine - The inverse of the cosine function, denoted by $\cos^{-1} x$ or $\arccos x$

Note: $cos^{-1}x \neq \frac{1}{cos\, x}$

Arcsine - The inverse of the sine function, denoted by $\sin^{-1} x$ or $\arcsin x$

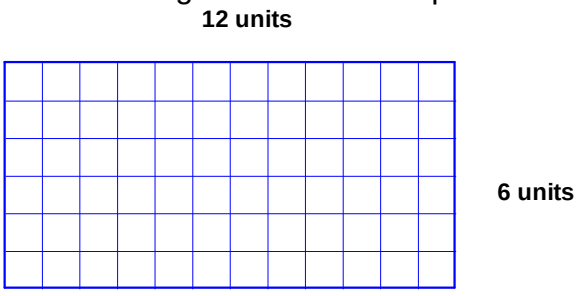
Note: $sin^{-1}x \neq \frac{1}{\sin x}$

Arctangent - The inverse of the tangent function, denoted by $\tan^{-1} x$ or $\arctan x$

Note: $\tan^{-1} x \neq \frac{1}{\tan x}$

Area of a polygon - The unique real number assigned to any polygon which indicates the number of non-overlapping square units contained in the polygon's interior

> **Example:** The area of the rectangle shown is 72 square units



Area=72 square units

Argument - The communication, in verbal or written form, of the reasoning process that leads to a valid conclusion.

Arithmetic sequence - A set of numbers in which the common difference between each term and the preceding term is constant

> **Example:** In the arithmetic sequence 2, 5, 8, 11, 14, … the common difference between each term and the preceding term is 3

Arithmetic series - The indicated sum of an arithmetic sequence.

Example: 1 + 3 + 5 + 7 + … is the indicated sum of the positive odd integers

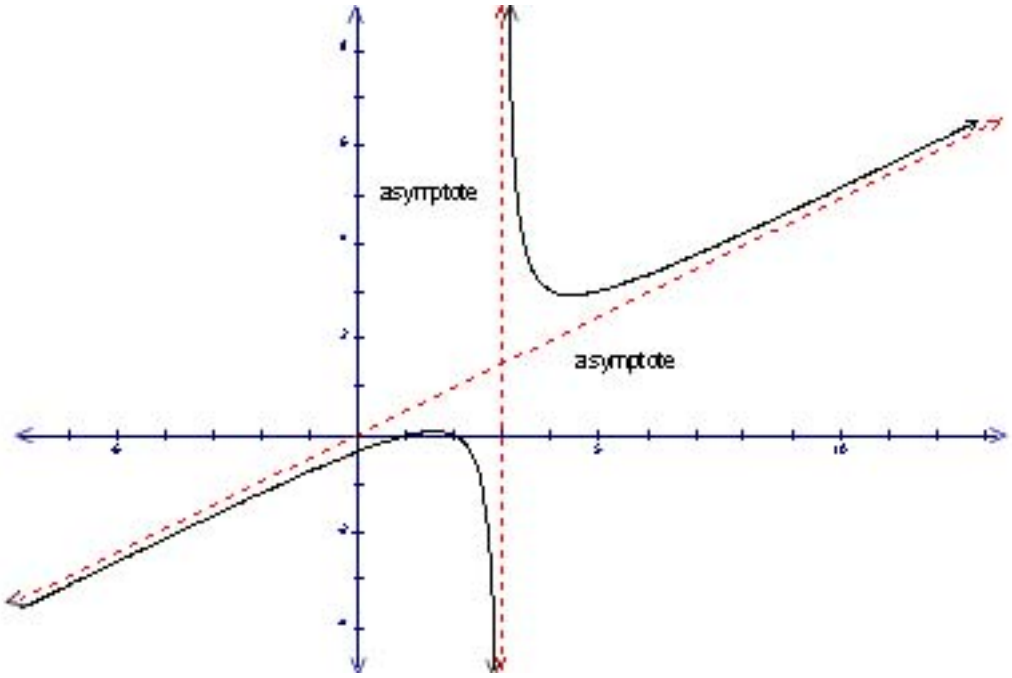
Array - A set of objects or numbers arranged in an order, usually in rows and columns or a matrix

Associative property - A property of real numbers that states that the sum or product of a set of numbers or variables has the same value, regardless of how the numbers or variables are grouped.

Examples: Addition: $2x + (3.5y + 1.3a\,z) = (2x + 3.5y) + 1.3z$
Multiplication: $-6a \times (18b \times 7c) = (-6a \times 18b) \times 7c$

Asymptote - A straight line or curve that is the limiting value of a curve

Example:

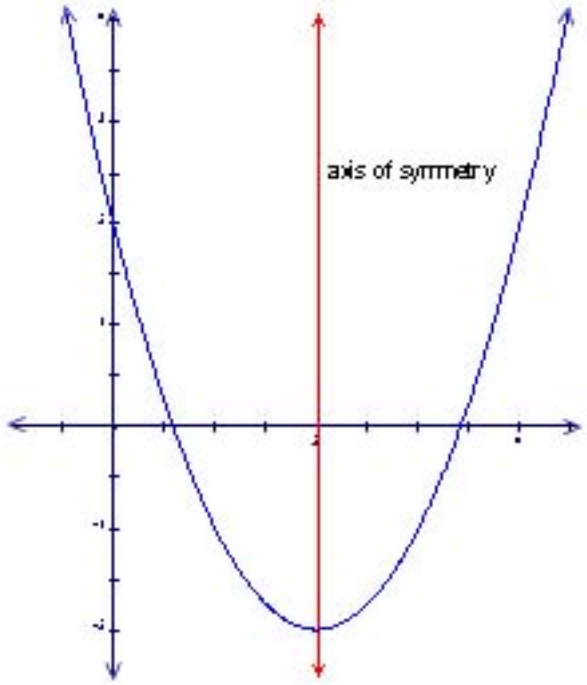


Axiom - A statement that is accepted without proof

Axis - A horizontal or vertical line used in the Cartesian coordinate system used to locate a point

Axis of symmetry - A line that divides a plane figure into two congruent reflected halves; Any line through a figure such that a point on one side of the line is the same distance to the axis as its corresponding point on the other side

Example:



This is a graph of the parabola
$y = x^2 - 4x + 2$ together with its axis of symmetry $x = 2$.

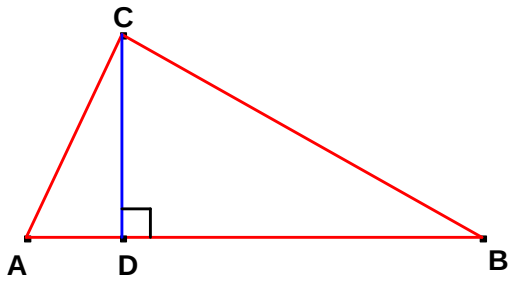
B

Base - A number or an expression that is raised to a power.

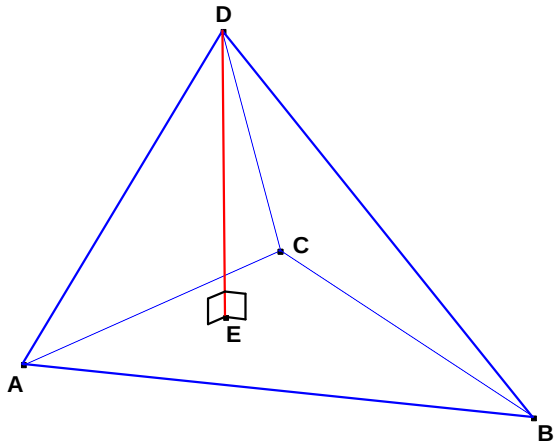
Example: $x+2$ is the base in the expression $(x+2)^3$ and 5 is the base in the expression 5^y

Base - Any side or face of a geometric figure to which an altitude is drawn.

Examples: For ΔABC: $\overline{CD}$ is an altitude and $\overline{AB}$ is a base



For pyramid $ABCD$: $\overline{DE}$ is an altitude and ΔABC is a base



Base of a logarithmic function - The number *b* in the logarithmic function $\log_b x = y$ where $b > 0$ and $b \neq 1$ if and only if $b^y = x$.

Base of an exponential function - The number *b* in the exponential function $y = ab^x$, where $a \neq 0, b > 0,$ and $b \neq 1$

Bernouli experiments - Probability experiments that can be described in terms of just two outcomes; an experiment that meet the following conditions: the experiment consists of *n* trials whose outcomes are either successes or failures, and the trials are identical and independent with a constant probability of success, *p*, and a constant probability of failure, *q* = 1- *p*.

Betweeness - A point B is between points A and C if and only if AB + BC = AC

Example:

Biased sample - A sample having a distribution that is determined not only by the population from which it is drawn, but also by some property that influences the distribution of the sample

Example: A poll to determine whether a stop sign is needed at a school crossing might be biased if the sample polled consisted only of parents who dropped their children off at school.

Biconditional - A statement formed by the conjunction of a conditional statement and its converse; a statement that can be written in "if and only if" form; a definition can always be written as a biconditional statement

Examples:

a) $(p \to q) \wedge (q \to p)$ is the biconditional of p and q and is written $p \leftrightarrow q$.

b) An angle is a right angle if and only if it has a measure of 90 degrees. This biconditional statement is equivalent to the following two statements: If an angle is a right angle, then it has a measure of 90 degrees and if an angle has a measure of 90 degrees, then it is a right angle.

Bimodal - A data set that has two modes

Binomial - An algebraic expression consisting of two terms

Examples:

$5a + 6$
$x^2 + 3y$
$9m - 13p$

Binomial expansion - The expansion of some power of a binomial expression

Example: The expansion of $(x+y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$

Binomial probability formula - A formula for determining the probability of a Bernoulli experiment

Binomial theorem - A method for expanding a binomial expression raised to some power

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{n-1}ab^{n-1} + \binom{n}{n}b^n = \sum_{k=0}^{n}\binom{n}{k}a^{n-k}b^k$$

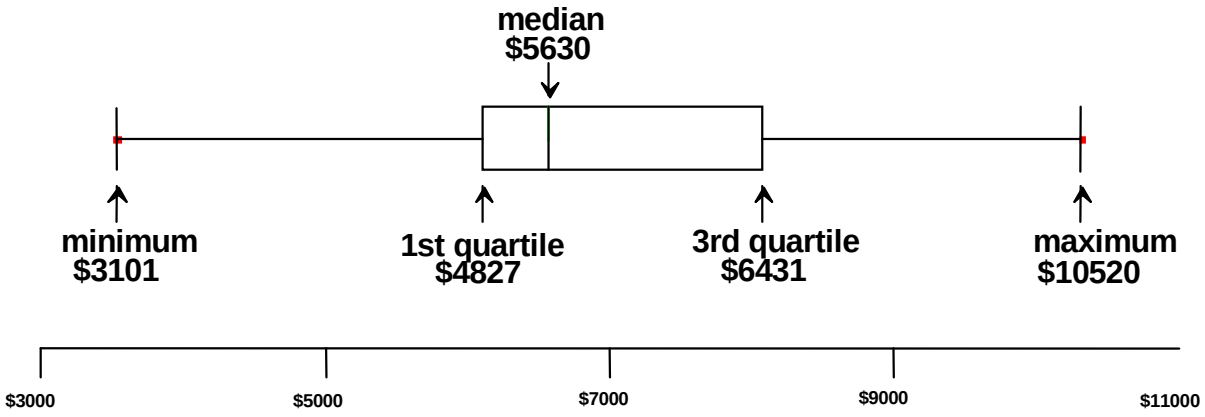
$$(a+b)^n = {}_nC_0a^n + {}_nC_1a^{n-1}b + {}_nC_2a^{n-2}b^2 + \ldots + {}_nC_{n-1}ab^{n-1} + {}_nC_nb^n = \sum_{k=0}^{n}{}_nC_ka^{n-k}b^k$$

Bivariate data - Data involving two variables

Box-and-whisker plot - A visual display of a set of data showing the five number summary: minimum, first quartile, median, third quartile, and maximum. This plot shows the range of scores within each quarter of the data. It is very useful for examining the variation in a set of data and comparing the variation of more than one set of data.

Example:

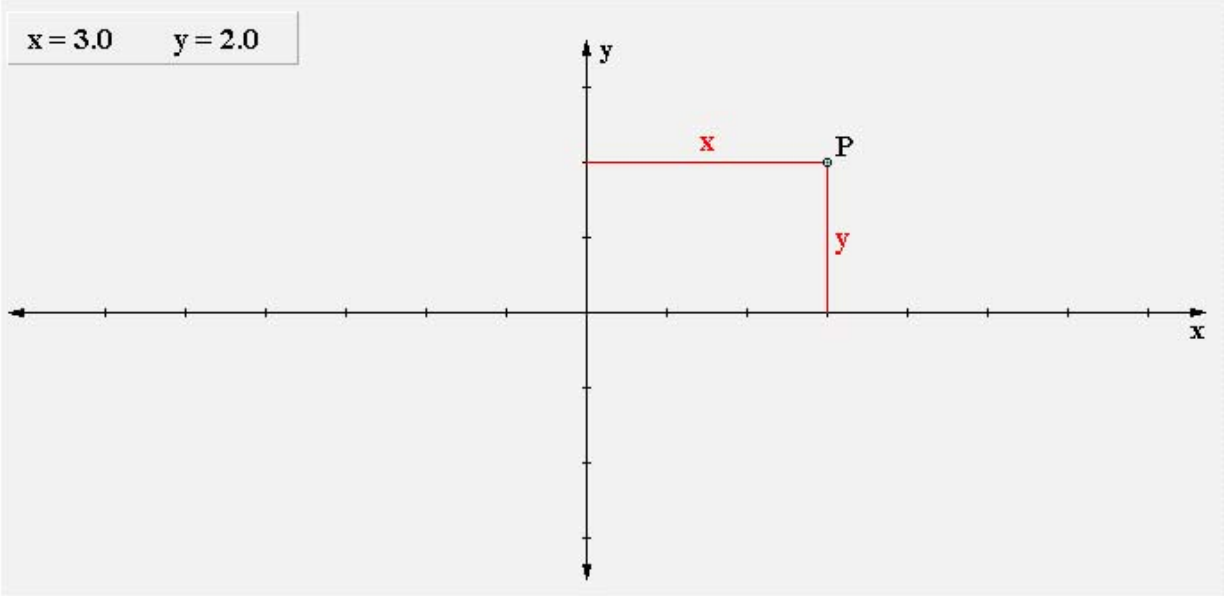
Annual food expenditures per household in the U.S. in 2005



C

Cartesian coordinates - An ordered pair of real numbers that establishes the location or address of a point in a coordinate plane using the distances from two perpendicular intersecting lines called the coordinate axes.

Example: Point P is identified by ordered pair (3, 2)



Cartesian plane -The set of all points in a plane designated by their Cartesian coordinates

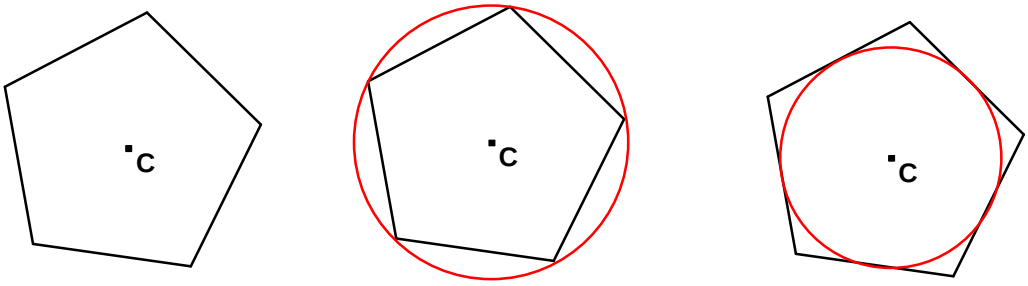
Center of dilation - A fixed point in the plane about which all points are expanded or contracted; the only invariant point under dilation

Center of gravity - The balance point of an object

Example: In a triangle, the center of gravity is the point of concurrency of the triangle's medians. This point is also called the centroid

Center of a regular polygon - The center of the circle which circumscribes or inscribes a regular polygon

Examples:



Circumscribes Inscribes

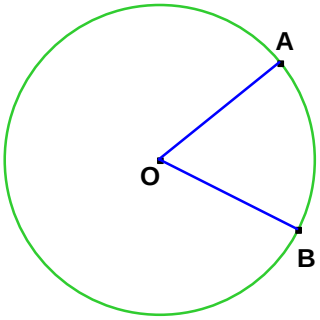
Center of a rotation - A fixed point in the plane about which all points are rotated

Center-radius equation of a circle - The form of the equation of a circle with center (*h, k*) and radius *r* given by the formula $(x-h)^2+(y-k)^2=r^2$

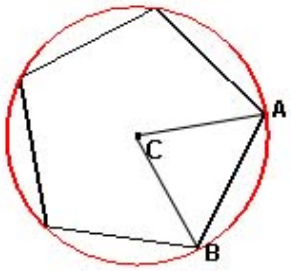
> **Example:** If the coordinates of the center of the circle are $(3,-4)$ and the length of the radius is 5, then the equation of the circle is $(x-3)^2+(y+4)^2=5^2$

Central angle - An angle in a circle whose sides are radii of the circle and whose vertex is the center of the circle; a central angle is equal in degrees to the arc it intercepts

> **Example:** Central angle *AOB*



Central angle of a regular polygon - An angle in a regular polygon with vertex at the center of the polygon and sides that are radii of its circumcircle



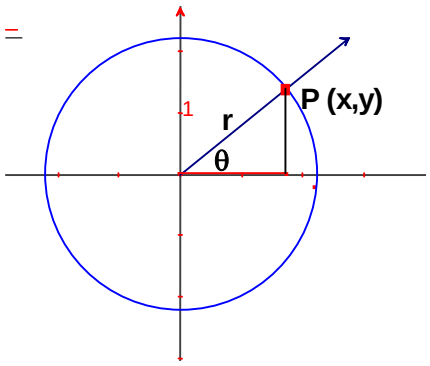
Centroid - is the term given to the center of an area, region, or polygon; the point of concurrency of the medians of a triangle; the center of mass of an object of uniform density

Chord - A line segment joining any two points on a circle; the diameter is the largest chord of a circle

Circle - The set of all points (or locus of points) in a plane that are a fixed distance, (called the radius) from a fixed point, (called the center)

Circular function - A function that relates the coordinates of a point on a circle to the distance from the point to the origin; these functions are called circular functions because the coordinates of a point on a circle are related to the line values of trigonometric functions

Example:



$$\cos\theta = \frac{x}{r},\ \sin\theta = \frac{y}{r},\ r = \sqrt{x^2 + y^2}$$

Circumcenter - The center of the circle circumscribed about a polygon; the point that is equidistant from the vertices of any polygon

Circumcircle - A circle that passes through all of the vertices of a polygon; also called a circumscribed circle

Circumference - The length of or distance around a circle. The formula for circumference is:
$C = 2\pi r = \pi d$

Clockwise - The direction in which the hands of a clock move around the dial; used to indicate the orientation of a transformation

Closure - A set "S" and a binary operation "*" are said to exhibit closure if applying the binary operation to any two elements in "S" produces a value that is a member of "S"

Coefficient - The numerical factor of a term in a polynomial

Example: 14 is the coefficient in the term $14x^3y$

Cofunction - The trigonometric function based on the *complement* of an angle; the value of a trigonometric function of an angle equals the value of the cofunction of the complement of the angle

Example:

Cofunctions, radians		**Cofunctions, degrees**	
$\sin\left(\frac{\pi}{2} - x\right) = \cos x$	$\cos\left(\frac{\pi}{2} - x\right) = \sin x$	$\sin(90° - x) = \cos x$	$\cos(90° - x) = \sin x$
$\tan\left(\frac{\pi}{2} - x\right) = \cot x$	$\cot\left(\frac{\pi}{2} - x\right) = \tan x$	$\tan(90° - x) = \cot x$	$\cot(90° - x) = \tan x$
$\sec\left(\frac{\pi}{2} - x\right) = \csc x$	$\csc\left(\frac{\pi}{2} - x\right) = \sec x$	$\sec(90° - x) = \csc x$	$\csc(90° - x) = \sec x$

Collinear points - Points that lie on the same line

Combination - An arrangement of objects in which order in not important; a collection of objects

Common base(s) - Exponential expressions or equations that have the same or equivalent bases

Example:

a) 2 is the common base in 2^3 and 2^4.
b) In the equation $3^x = 3^2$, the common base is 3.

Common difference - The difference between any two successive terms of an arithmetic sequence

Common factor - A number, polynomial, or quantity that divides two or more numbers or algebraic expressions evenly

Example:

1, 3, 5, 15, are common factors of 15 and 30
$2x$ is a common factor of $4xy$ and $6x^2$
$x-2$ is a common factor of x^2-x-2 and x^2-6x+8

Common logarithm - A logarithm to base 10; the common logarithm of x is written log x; For example, log 100 is 2 since $10^2 = 100$

Common ratio - The ratio of any two successive terms of a geometric sequence

Common tangents - Lines that are tangent to two or more circles

Commutative property - A property of real numbers that states that the sum or product of two terms is unaffected by the order in which the terms are added or multiplied; i.e., the sum or product remains the same; independent of order; as in "a x b = b x a"; subtraction and division are not commutative for the set of real numbers

Examples: Addition: $-2x + 3.5y = 3.5y + -2x$
Multiplication: $xy^2 = y^2x$

Compare - To state the similarities or differences between two or more numbers, objects, or figures by considering attributes such as size, shape, odd, even

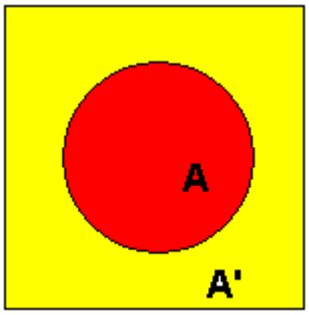
Complementary angles - Two angles the sum of whose measures is 90 degrees

Complement of a set - The elements of a universe not contained in a given set; the subset that must be added to any given subset to yield the original set. The complement of set A is indicated by A' or A^C

Universe = interior of the square

Set A= the circular region

Complement of A is A' (or A^C)



Completing the square - A process used to change an expression of the form ax^2+bx+c into a perfect square binomial by adding a suitable constant

Complex fraction - A fraction with another fraction in its numerator, denominator, or both

Example: $\dfrac{\frac{2}{3x}}{\frac{1}{x}}$ or $\dfrac{x-\frac{1}{x}}{x+1}$

Complex number - Any number that can be expressed in the form $a+bi$, where a and b are real numbers and i is the imaginary unit

Composition of functions - A way of combining functions in which the output of one function is used as the input of another function; the formation of a new function h from functions f and g using the rule $h(x)=g\circ f(x)=g\left[f(x)\right]$ for all x in the domain of f for which $f(x)$ is in the domain of g

Compound event - An event that is derived from two or more simple events

> **Example:** If we roll two dice, then the event "getting a six on either the first or second die" is a compound event.

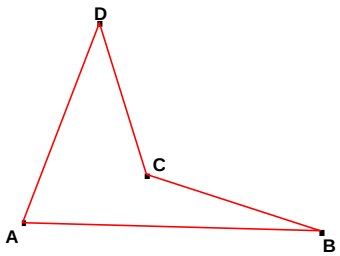
Compound interest - A method of calculating interest in which interest earned is added to the principal and thereafter also earns interest

Compound locus - A set of points satisfying two or more locus conditions

Compound statement - A statement formed from two or more simple statements using the logic connectives, *or*, *and*, *if...then*, or *if and only if*

Concave polygon - A polygon that has at least one diagonal outside the polygon

> **Example**: concave polygon



Concentric circles - Two or more circles having the same center and different radii

Conclusion - An answer or solution arrived at through logical or mathematical reasoning; the "*then*" clause in an "*if-then"* statement; the final statement in a proof which follows logically from previous true statements

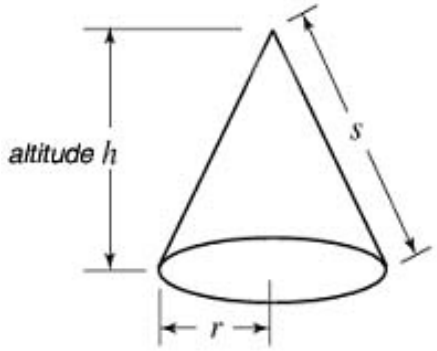
Concurrence - The concept of three or more lines intersecting in a single (common) point; having a single point of intersection; the medians of a triangle are concurrent

Conditional probability - A probability that is computed based on the assumption that some event has already occurred; the probability of event B given that event A has occurred is written P (B|A)

Conditional statement - A statement formed from two given statements by connecting them in the form if..., then...

> **Example:** "If p then q" is a conditional statement (*p* implies *q*) and is denoted $p \rightarrow q$; p is called the hypothesis and q is called the conclusion

Cone - A solid formed by a circular region (the base) and the surface formed by the straight line segments connecting points on the boundary of the base with a fixed point (the vertex) not in the plane of the base

> **Example:**
>
> 
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Conjecture - An educated guess; an unproven hypothesis based on observation, experimentation, data collection, etc.

Conjugate - Two binomials whose first terms are equal and last terms are opposites

> **Example:** $a+b$ and $a-b$, $2+3\sqrt{5}$ and $2-3\sqrt{5}$, $3+2i$ and $3-2i$

Conjunction - A compound statement formed using the word "*and"*; a conjunction is true only if both clauses are true

> **Example:** Today is Tuesday and the sun is shining.

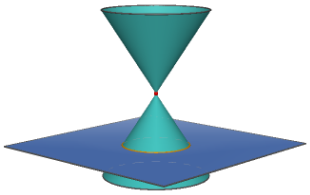
Congruent - Having the same size and shape

> **Examples:** Two *line segments* are congruent if they have the same length.

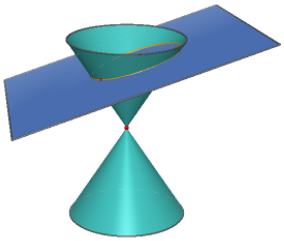
Two *angles* are congruent if the have the same measure.
Two *polygons* are congruent if there exists a one-to-one correspondence between their vertices such that their corresponding sides are congruent and their corresponding angles are congruent

Conic sections – The curves that are formed by the intersection of a plane and a right circular cone

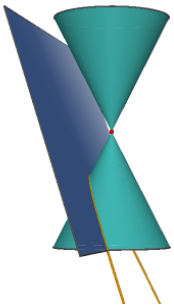
Example: a parabola, a circle, an ellipse, or a hyperbola:



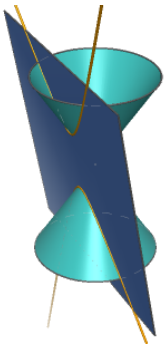
circle



ellipse



parabola



hyperbola

Consistency - A property of an axiomatic system where no axiom(s) can be used to contradict any other axiom(s)

Constant function - A function where each element of the domain is mapped to the same element in the range; the graph of a constant function is a horizontal line

Example: $f(x) = 4$

Constant of proportionality - The number representing the ratio of any two corresponding sides in two similar geometric figures

Construct - To draw a figure using only a compass and a straightedge

Constraints - Any restriction placed on the variables in a problem

Contradiction - of words or propositions so related that both cannot be true and both cannot be false; "`perfect' and `imperfect' are contradictory terms"; a proposition is either true of false, but not both

Contrapositive of a statement - A statement formed by interchanging the hypothesis and conclusion of a conditional statement and negating each clause

Example: $\sim q \sim p$ is the contrapositive of $p\ q$

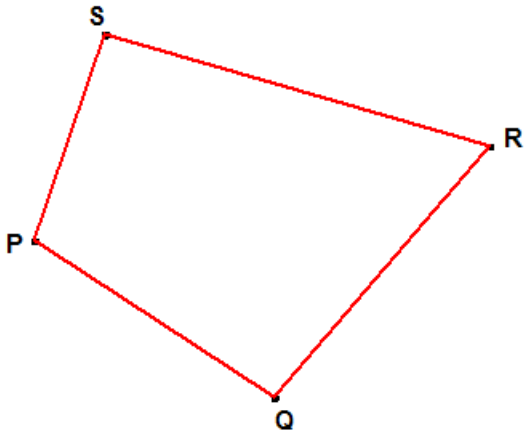
Controlled experiment - An experiment which compares the results obtained from an experimental sample against a control sample

Converse of a statement - A statement formed by interchanging the hypothesis and conclusion of a conditional statement

Example: $q \to p$ is the converse of $p \to q$

Convex polygon - A polygon is convex if a line segment connecting any two points of the polygon lies entirely in the polygon's interior

Convex polygon



Coordinates - An ordered pair of numbers that identifies a point on a coordinate plane, written as (*x*, *y*); the number represented by "*x*" is called the x-coordinate (abscissa); the number represented by "*y*" is called the y-coordinate (ordinate)

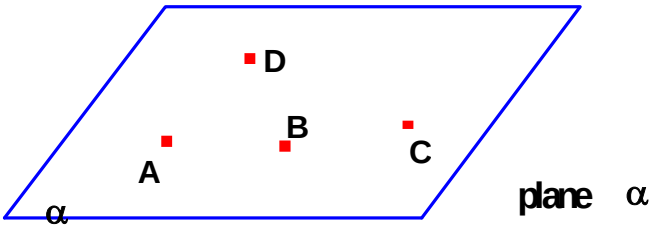
Example: In Cartesian coordinates $(2,3)$ the number 2 is the x-coordinate (abscissa) and 3 is the y-coordinate (ordinate)

Coordinate geometry - An approach to geometry in which a point is represented by coordinates and algebraic methods of reasoning are used; also called analytical geometry

Coordinate plane - The set of all points in a plane designated by their Cartesian coordinates; also called the Cartesian plane

Coplanar - Any three or more points that lie in the same plane

Example: Points *A, B, C,* and *D* are coplanar



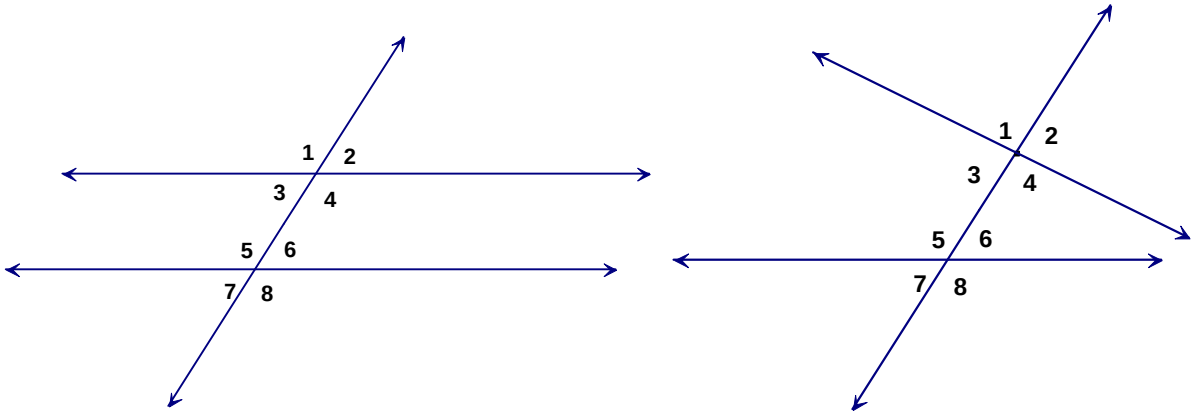
Correlation - A statistical measure that quantifies how pairs of variables are related; a linear relationship between two variables

Correlation coefficient - A number, r, between -1 and 1 that indicates the strength and direction of the linear relationship between two sets of numbers

Example: If r=1, then the points lie on a line which has a positive slope and the two sets of numbers are said to be in perfect positive correlation. If r=-1, then the points lie on a line which has a negative slope and the two data sets are said to be in perfect negative correlation

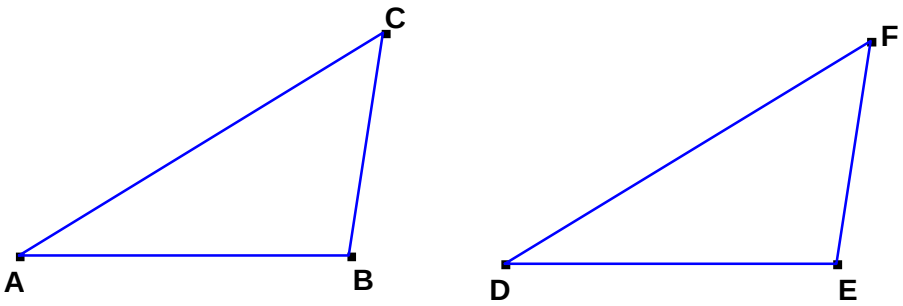
Corresponding angles - A set of angles formed on each of two or more lines cut by a transversal that are in the same position relative to each line and to the transversal. Note: As illustrated in the second example, the lines do *not* need to be parallel

Examples: In the both diagrams below, $\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 7$, $\angle 4$ and $\angle 8$ are pairs of corresponding angles



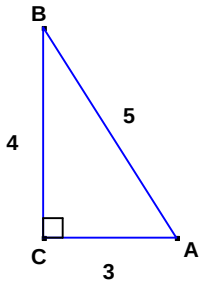
Corresponding parts - In two geometric figures, the points, sides, and/or angles which are in the same relative position when the figures are placed in a one-to-one correspondence

Example: In the one-to-one correspondence implied by $\Delta ABC \cong \Delta DEF$, point A corresponds to point D; side $\overline{AB}$ corresponds to side $\overline{DE}$; and $\angle ABC$ corresponds to $\angle DEF$, etc.



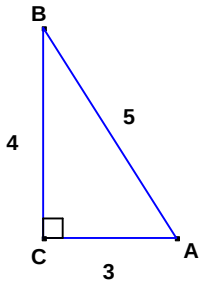
Cosecant - For a given acute angle θ in a right triangle, the ratio of the length of the hypotenuse of the triangle to the side opposite the acute angle; Also the reciprocal of the sine ratio of the given angle, written as $csc\,\theta$

Example: In this right triangle, $\csc A = \frac{5}{4}$ and $\csc B = \frac{5}{3}$



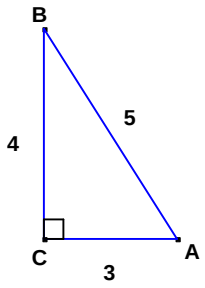
Cosine - For a given acute angle θ in a right triangle, the ratio of the length of the side adjacent to an acute angle to the length of the hypotenuse; the cosine of an angle is written as $cos\,\theta$

Example: In this right triangle, $\cos A = \frac{3}{5}$ and $\cos B = \frac{4}{5}$



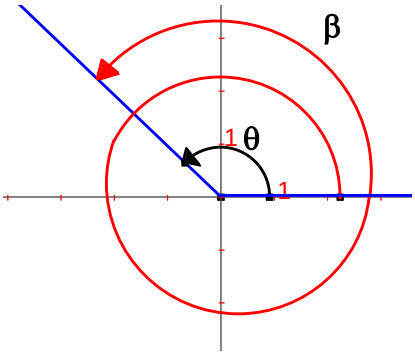
Cotangent - For a given acute angle θ in a right triangle, the ratio of the side adjacent an acute angle to the side opposite the acute angle; the cotangent is the reciprocal of the tangent ratio of the given angle, written as $cot\,\theta$

Example: In this right triangle, $\cot A = \frac{3}{4}$ and $\cot B = \frac{4}{3}$

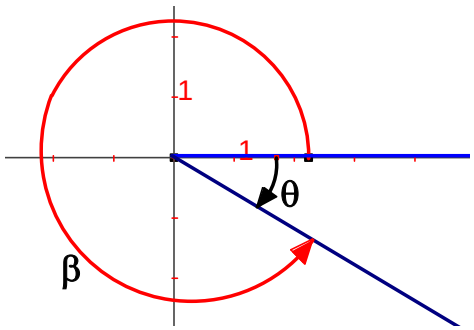


Coterminal angles - Angles in standard positions that share the same terminal side; formed by different rotations that have the same initial and terminal sides

Example: In each figure below, θ and β are coterminal angles



The measure of $\theta = 135^\circ$ and the measure of $\beta = 495^\circ$



The measure of $\theta = -45^\circ$ and the measure of $\beta = 315^\circ$

Counterclockwise - The direction opposite the way in which the hands of a clock move around the dial; used to indicate the orientation of a rotation

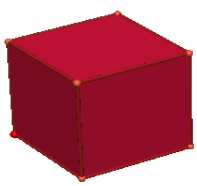
Counterexample - An example that disproves a general statement

Example: The statement that the sum of two numbers is less than their product could be refuted by the counterexample that $2+1>2\cdot 1$.

Cross-section - A plane section perpendicular to the longest axis of a solid

Cube - A polyhedron with six square faces; a cube (or hexahedron) is one of the five platonic solids

Example:



Cubic unit - A unit for measuring volume

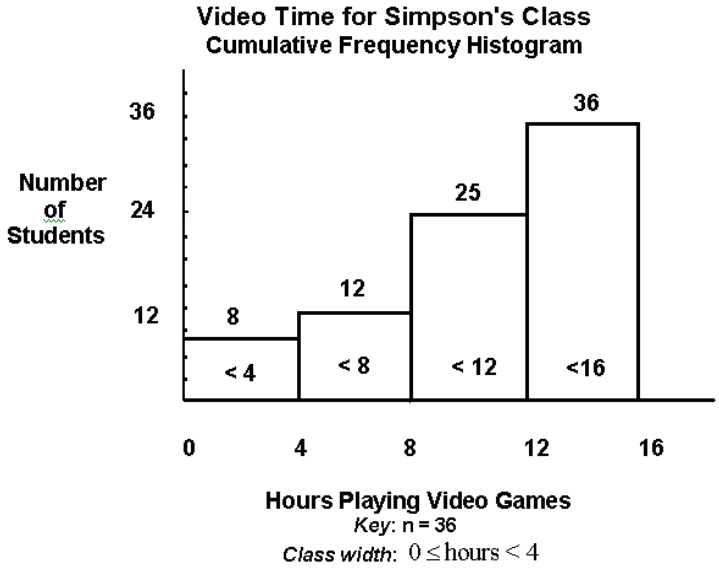
Cumulative frequency table - A table that shows how often each item, number, or range of numbers occurs in a set of data. This table displays the total number of scores that fall into each of several cumulative intervals. The cumulative intervals are created by adding the preceding tallies (of lower scores) to the new tallies for each interval:

Example: 5 , 7, 6 , 8 , 9 , 5 , 13 , 2 , 1 , 6 , 5 , 14 , 10 , 5 , 9

Interval	Frequency	Cumulative Frequency
1-5	6	6
1-10	7	13
1-15	2	15

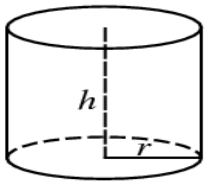
Cumulative frequency histogram - A histogram where each bar contains all the data up to and including the data in that bar's interval

Example: This histogram shows the number of students in Mrs. Simpson's class that are in each interval. The students were asked how many hours they spent playing video games in one week.



Cylinder - A solid geometric figure bounded by two parallel bases which are congruent circles and a lateral surface which consists of the union of all line segments joining points on each of those circles

Example:



D

Decagon - A polygon with ten sides

Deductive proof - A formal proof based on logical argument that is justified using axioms and/or theorems

Deductive reasoning - A process of showing that certain statements follow logically from agreed upon assumptions and proven facts; a system of reasoning which proceeds from the general to the specific; any deductive proof can be arranged in a pattern, called a syllogism, in which there are three steps:

1. A general statement
2. A specific statement satisfying the hypothesis of the general statement
3. A conclusion based on the general and specific statements

The general statement is the major premise and the specific statement is the minor premise:

> **Example:** All humans are mortal. Socrates is human.
> Therefore, Socrates is mortal.

Degree measure - A unit of angle measure equal to $\frac{1}{360}$ of a complete revolution

Degree of a monomial - The sum of the exponents of the variables in the monomial

> **Example:** The degree of the monomial $4x^3$ is three
> The degree of the monomial x^2y^3 is five
> The degree of xy is two
> The degree of 7 is zero

Degree of a polynomial - The highest degree of any monomial term in the polynomial

Dependent events - Two events in which the outcome of the first event affects the outcome of the second event

> **Example**: The probability of choosing a king on the second draw is dependent on whether or not a king was chosen on the first draw if the card was not replaced

Dependent variable - A variable whose value is determined by a second variable

Derivative of a function – the instantaneous rate of change of a function with respect to a variable; the derivative of a function $f(x)$ at a point a is the limit of the difference quotient as *x* approaches *a*; the derivative of $y = f(x)$ is usually denoted by $f'(x)$ or $\frac{dy}{dx}$

Determinant – a square array of numbers; if a determinant has two rows and two columns, it is called a determinant of the second order. The value of this determinant is the difference of the products of its diagonals:

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

The value of this determinant is the difference of the products of its diagonals: $ad - bc$

Diagonal - A line segment that connects two non-consecutive vertices of a polygon

Diameter - A chord of the circle that passes through the center of the circle

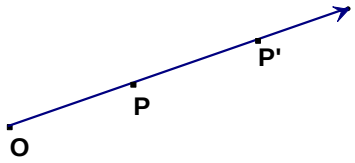
Diameter of a sphere - A line segment that connects two points on the surface of a sphere and that passes through the center of the sphere

Difference of two perfect squares - A binomial of the form $a^2 - b^2$ which can be factored into $(a - b)(a + b)$

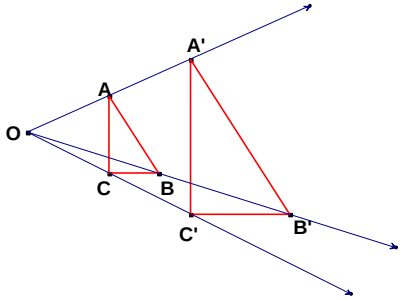
Dihedral angle - An angle formed by two interesting planes

Dilation - A transformation of the plane such that if *O* is a fixed point, *k* is a non-zero real number, and *P*' is the image of point *P*, then *O*, *P* and *P*' are collinear and $\frac{OP'}{OP} = k$

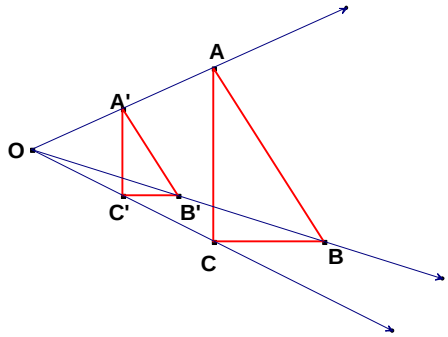
Examples:



P' is the image P under a dilation about O of ratio 2;
$OP' = 2OP$



$\Delta A'B'C'$ is the image of ΔABC under a dilation about O of ratio 2;
$OA' = 2OA$, $OB' = 2OB$, and $OC' = 2OC$



$\Delta A'B'C'$ is the image of ΔABC under a dilation about O of ratio ½;
$OA' = \frac{1}{2}OA$, $OB' = \frac{1}{2}OB$, and $OC' = \frac{1}{2}OC$.

Direct transformation - Any transformation of the plane that preserves orientation; a translation, rotation or dilation

Direct variation - A relationship in which the ratio of two variables is constant; a direct variation has an equation in the form $y = kx$, where x and y are variables and k is the constant of variation; directly proportional - proportional to a constant multiple of an independent variable

Discriminant - The expression $b^2 - 4ac$ where a, b and c are the coefficients of the quadratic equation $ax^2 + bx + c = 0$; the discriminant is used to determine the nature of the roots of the quadratic equation:

1. If $b^2 - 4ac = 0$ the roots are real, rational, and equal
2. If $b^2 - 4ac$ is positive and a perfect square, the roots are real, rational, and equal
3. If $b^2 - 4ac$ is positive and not a perfect square, the roots are real, irrational, and unequal
4. If $b^2 - 4ac$ is negative, the roots are complex numbers

Distance between two points - The length of the line segment joining the two points; a unique non-negative real number

Distance between a point and a line - The length of the perpendicular segment from the point to the line

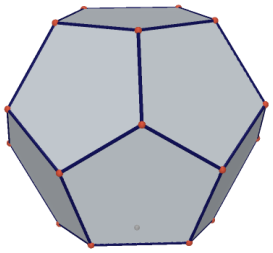
Distance between two parallel lines - The length of a line segment drawn from any point on one line perpendicular to the second line

Distributive property - A property of real numbers that states that the product of a number and the sum or difference of two numbers is the same as the sum or difference of their products; $a(b \pm c) = ab \pm ac$

Example: Multiplication over addition: $2(15 + 4) = 2 \times 15 + 2 \times 4$
Multiplication over subtraction: $4(12 - 8) = 4 \times 12 - 4 \times 8$

Dodecahedron - A polyhedron that has twelve faces; a regular dodecahedron is one of the five Platonic solids and has twelve regular pentagons as faces

Example: dodecahedron



Domain - The set of values of the independent variable for which a given function is defined; the set of first coordinates in the ordered pairs of a relation.

Double and half angle formulas for trigonometric functions - Formulas used to determine trigonometric values for double or half of a given angle:

$$\sin 2A = 2\sin A\cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$\cos 2A = 2\cos^2 A - 1$$
$$\cos 2A = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$$

$$\sin\frac{1}{2}A = \pm\sqrt{\frac{1-\cos A}{2}}$$
$$\cos\frac{1}{2}A = \pm\sqrt{\frac{1+\cos A}{2}}$$
$$\tan\frac{1}{2}A = \pm\sqrt{\frac{1-\cos A}{1+\cos A}}$$

Double root - A root of an equation that occurs twice; a value r is a double root of an equation $f(x)=0$ if $(x-r)^2$ is a factor of $f(x)$

E

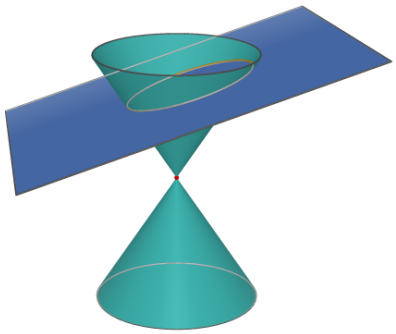
e - The base of the natural logarithm; a number commonly encountered when working with exponential functions to model growth, decay, continuously compounded interest; $e \approx 2.7182818284....$

Edge of a polyhedron - A line segment that connects two consecutive vertices of a polyhedron.

Element - An object contained in a set

Ellipse - a closed plane curve resulting from the intersection of a circular cone and a plane cutting completely through it; the sums of the distances from the foci to any point on an ellipse is constant; a set of points P_1 in a plane, such that the sum of the distances from P to two fixed points F_1 and F_2 is a given constant k;

Example: $PF_1 + PF_2 = k$



Empirical probability - An estimate of the probability of an event based on the results of repeated trials of the event

Endpoint - A point at either the end of a line segment, or arc, or the initial point of a ray

Equation - a mathematical statement that two expressions are equal

Equiangular - A polygon with all interior angles congruent

Equidistant - the same distance apart at every point

Equilateral polygon - A polygon with all sides congruent

Equilateral triangle - A triangle with three congruent sides; equilateral triangles are also equiangular

Equivalence relation - A relation that exhibits the *reflexive*, *symmetric*, and *transitive* properties

Example: Triangle congruence is an equivalence relation since it is:

reflexive: $\Delta ABC \cong \Delta ABC$,

symmetric: If $\Delta ABC \cong \Delta DEF$ then $\Delta DEF \cong \Delta ABC$, and

transitive: If $\Delta ABC \cong \Delta DEF$ and $\Delta DEF \cong \Delta GHI$ then $\Delta ABC \cong \Delta GHI$

Equivalent forms - Different ways of writing numbers or expressions that have equal values

Example: $\frac{2}{3}$ is equivalent to $\frac{8}{12}$

$3 \cdot 3 \cdot 5 \cdot 5 \cdot 5 \cdot 5$ is equivalent to $3^2 \cdot 5^4$

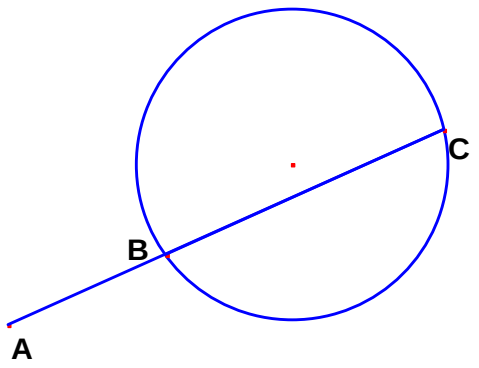
Euclidean Geometry – the geometry based on Euclid's *Elements*, a treatise containing 13 books, in which Euclid summarized and organized the mathematical knowledge of his time (c.a. 330 B.C.). The *Elements* presented the mathematical knowledge as a postulational system made up of undefined and defined terms, axioms, and theorems.

Euclidean Parallel Postulate - Any assumption equivalent to the following statement: If l is any line and P is any point not on l, then there exists exactly one line through P that is parallel to l

Euler line - named after Leonhard Euler, a line determined from any triangle that is not equilateral; it passes through the orthocenter, the circumcenter, the centroid, and the center of the nine-point circle of the triangle; Euler showed in 1765 that in any triangle, the orthocenter, circumcenter and centroid are collinear

External segment of a secant - If a secant is drawn to a circle from an external point, the portion of the secant that lies outside the circle

Example:



$\overline{AB}$ is the external segment for secant $\overline{AC}$

Exact answer - The solution to an equation that has not been rounded

Example: When solving the equation $x^2 = 3$, the exact answer is $x = \pm\sqrt{3}$; an answer such as 1.73205, although correct to five decimal places, has been rounded and thus is not an exact answer

Exact value - The value of an expression that has not been rounded.

Example: The exact value of $\cos\left(\frac{\pi}{4}\right)$ is $\frac{\sqrt{2}}{2}$, while 0.7071 is an approximation

Expand a binomial - The process of creating a polynomial by raising a binomial to an integral power

Example: When expanded $(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

Experimental probability - A probability calculated by performing an experiment, rather than by analyzing a situation mathematically; the ratio of the number of times the event occurs to the total number of trials or times the activity is performed

Explicit formula - For a sequence $a_1, a_2, a_3, ...a_n...$, a formula that is used to generate the nth term of a sequence.

Exponent - A number that tells how many times the base is used as a factor of a term; in an expression of the form b^n, n is called the exponent, b is the base, and b^n is a power of b

Exponential decay - The decreasing exponentially of a quantity over time represented by $y = ab^x$ where $a > 0$ and $0 < b < 1$

Example: $y = 5(0.1)^x$. Each time x is increased by 1, y decreases to one tenth of its previous value.

Exponential form - An expression or equation containing exponents

Example: The equation $3^2 = 9$ is the exponential form of the logarithmic equation $\log_3 9 = 2$. In exponential form, $32 = 2^5$

Exponential function - A function with a variable in the exponent; an equation in the form $y = ab^x$, where $a \neq 0$ and $b > 0, b \neq 1$

Example: $A = 3(1.02)^t$ is an exponential function

Exponential growth - The increasing exponentially of a quantity over time represented by $y = ab^x$ where $a > 0$ and $b > 1$

Example: $y = 5(2)^x$. Each time x is increased by 1, y increases by a factor of 2

Expression - A mathematical representation containing numbers, variables, and operation symbols; an expression does not include an equality or inequality symbol

Exterior of a geometric figure - The set of all points outside a geometric figure

Exterior angle of a polygon - An angle formed by a side of a polygon and the extension of an adjacent side

Extraneous root - A solution of a derived equation that is not a solution of the original equation; roots that do not check in the original equation. When fractional equations are multiplied by a common denominator or when radical expressions are squared, extraneous roots can be introduced because the resulting equation is not equivalent to the original equation. Therefore,

solutions must be checked to determine whether the root found is a solution of the original equation.

Extrapolate -The process of using a given data set to estimate the value of a function or measurement beyond the values already known.

F

Face of a polyhedron - Any one of the polygons that bound a polyhedron

Factor – (noun) a whole number that is a divisor of another number; an algebraic expression that is a divisor of another algebraic expression

Example: 3 is a factor of 12

Factor - (verb) Find the number of algebraic expressions that give an indicated product

Example: To factor $x^2 - x - 6$, write $(x-3)(x+2)$

Factor of proportionality - the constant value of the ratio of two proportional quantities *x* and *y*; usually written *y* = *kx*, where k is the factor of proportionality

Factorial - The product of a given integer and all smaller positive integers. n factorial is written as n! Note: 0! = 1 and 1! = 1

Example: 6! = 6·5·4·3·2·1 = 720
n! = *n*·(*n* – 1)·(*n* – 2) · · · 3·2·1

Five number summary - For a data set, these include the minimum, the first quartile, the median, the third quartile, and the maximum.

Fixed point - A point that is its own image under a transformation of the plane; for example, the center of a rotation or dilation; a point on the line of reflection

Fraction – a part of a unit or an indicated quotient of one number divided by another. If a line is divided into 4 equal parts, one part is said to be one fourth of the line and is written $\frac{1}{4}$. The number of parts into which any unit is divided is called the denominator of the fraction. The number of equal parts taken is called the numerator. For $\frac{1}{4}$, 1 is the numerator and 4 is the denominator.

Fractional exponent - An exponent that is a rational number

Example: $8^{\frac{2}{3}} = \sqrt[3]{8^2} = \left(\sqrt[3]{8}\right)^2 = 4$

Frequency table - A table that shows how often each item, number, or range of numbers occurs in a set of data; frequency means just that, how many times a given element repeats. It's easier to make a frequency table if we arrange the elements in ascending order:

Example: The data {5 ,7, 6 , 8 , 9 , 5 , 13 , 2 , 1 , 6 , 5 , 14 , 10 , 5 , 9} can be displayed as a frequency distribution. It's easier to make a frequency table if we arrange the elements in ascending order:

1, 2, 5, 5, 5, 5, 6, 6, 7, 8, 9, 9, 10, 13, 14

Then you count the number of times that each element occurs.

Interval	Frequency
1-5	6
6-10	7
11-15	2

Function - A rule that assigns to each number x in the function's domain a unique number $f(x)$. To be called a function, the rule must assign only one element of the range to each element of the domain. A rule that assigns more than one element of the range to each element of the domain is called a relation.

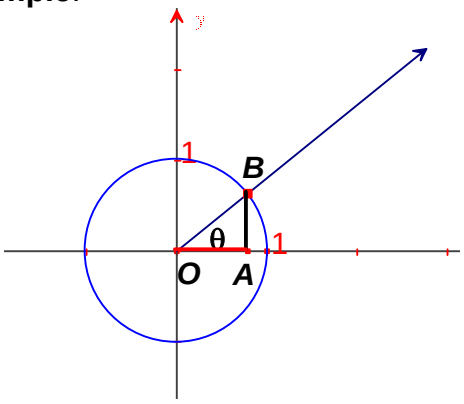
G

Geometric inequality - A statement in geometry which indicates that quantity is greater than another quantity; for example, in a triangle, the measure of an exterior angle is greater than the measure of either remote interior angle

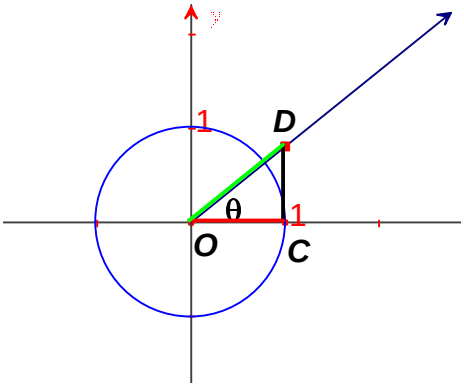
Geometric mean - The geometric mean, also called the mean proportional, of two numbers a and b is the square root of their product. If $\frac{a}{m} = \frac{m}{b}$ then m is the geometric mean of a and b

Geometric representation of the circular functions - The representation of circular functions on a circle of unit radius. The trigonometric functions are called circular functions because their values are related to the lengths of specific line segments associated with a circle of unit radius.

Example:



$OA = \cos\theta,\ AB = \sin\theta,\ OB = 1$



$CD = \tan\theta,\ OD = \sec\theta,\ OC = 1$

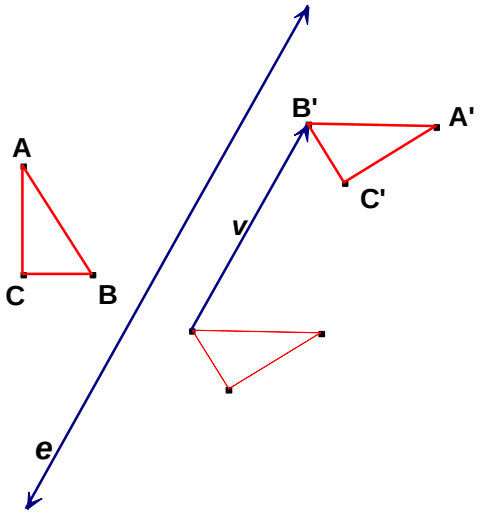
Geometric sequence - a sequence of numbers where each term after the first is found by multiplying the previous one by a fixed non-zero number called the common ratio. For example, the sequence 2, 6, 18, 54, ... is a geometric progression with common ratio 3. Similarly 10, 5, 2.5, 1.25, ... is a geometric sequence with common ratio 1/2

Geometric series - The indicated sum of a geometric sequence

Geometry - Branch of mathematics that deals with the properties, measurement, and relationships of points, lines, angles, surfaces, and solids

Glide reflection - A transformation that is the composition of a line reflection and a translation through a vector parallel to that line of reflection.

Example:



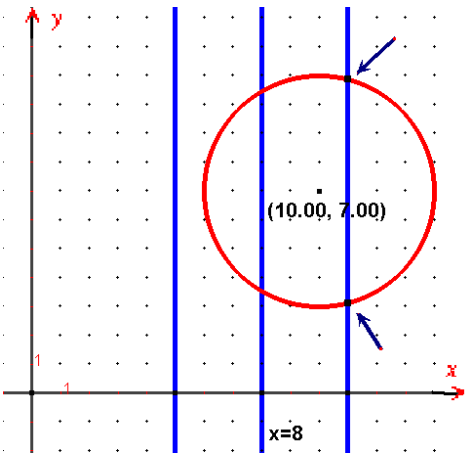
$\Delta A'B'C'$ is the image of ΔABC under a glide reflection that is the composition of a reflection over line *l* and a translation through vector *v.*

Golden ratio - When a line segment $\overline{AB}$ is divided by an interior point P such that $\frac{AB}{AP}=\frac{AP}{PB}$, the ratio $\frac{AB}{AP}=\frac{1}{2}\left(1+\sqrt{5}\right)$ is called the golden ratio; approximately 1.61803

Golden rectangle - A rectangle whose adjacent sides have a ratio equal to the golden ratio.

Graphical representation - A graph or graphs used to model a mathematical relationship.

Example: The figure below is a graphical representation of the locus of all points 4 units from A (10, 7) and 3 units from x = 8



Graphical solution of a system of equations - The set of points in the plane whose coordinates are solutions to a system of equations

Great circle - The intersection of a sphere with any plane passing through the center of the sphere; for example the equator is a great circle; all lines of longitude are great circles

Greatest common denominator (GCD) - the largest positive integer that divides the numbers without a remainder; for example, the GCD of 8 and 12 is 4; the GCD is useful for reducing

fractions to be in lowest terms; there are several methods of determining the GCD. One is by determining the prime factorizations of the numbers and comparing factors;

$2 \mid \underline{42, 60, 84}$

$3 \mid \underline{21, 30, 42}$ GCD = 2 x 3 = 6

$0 \mid \underline{7, 10, 14}$

Another method is the Euclidean algorithm, which uses the division algorithm in combination with the observation that the GCD of two numbers also divides their difference: divide 48 by 18 to get a quotient of 2 and a remainder of 12. Then divide 18 by 12 to get a quotient of 1 and a remainder of 6. Then divide 12 by 6 to get a remainder of 0, which means that 6 is the GCD.

Greatest common factor (GCF) - The greatest number or expression that is a factor of two or more numbers or expressions

Example: 12 is the GCF of 24 and 36.
$5xy$ is the GCF of $25x^2y$ and $10xy^2$

Growth factor - The base of an exponential function

Example: In the formula $f(t) = ab^t$ the base b, is the growth factor

H

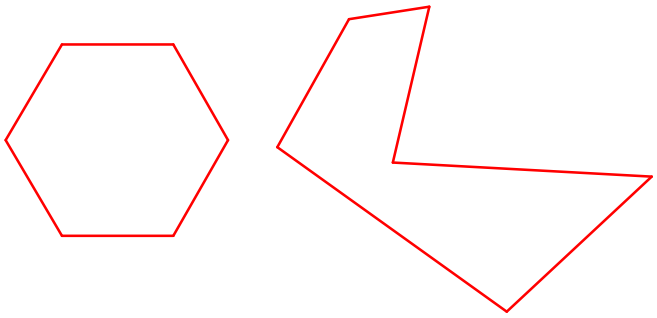
Half-life - The time needed for an amount of a substance to decrease by one-half

Hemisphere - Half of a sphere bounded by a great circle

Heron's formula - The formula expressing the area of a triangle, A in terms of its sides a, b and c; $A=\sqrt{s(s-a)(s-b)(s-c)}$ where $s=\frac{1}{2}(a+b+c)$ and is called the semi-perimeter

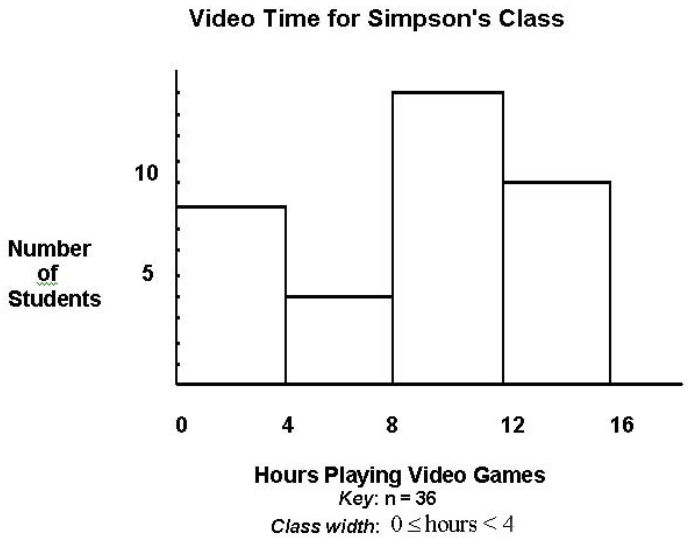
Hexagon - A polygon with six sides

Examples:



Histogram - A frequency distribution for continuous quantitative data. The horizontal axis is a number line that displays the data in equal intervals. The frequency of each bar is shown on the vertical axis.

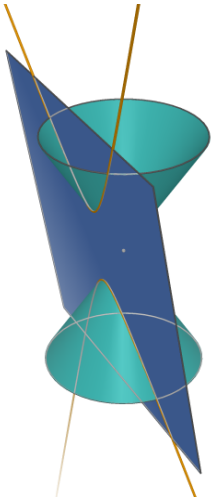
Example: This histogram shows the number of students in Simpson's class that are in each interval. The students were asked how many hours they spent playing video games in one week:



Horizontal-line test - A test using any horizontal line to determine whether or not a function is one-to-one; a function *f(x)* is one-to-one if and only if no horizontal line intersects the graph of f(x) more than once

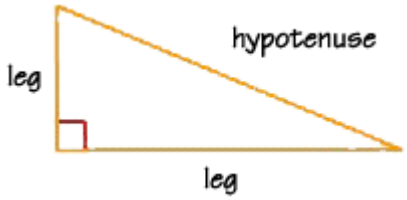
Hyperbola - Set of points P in a plane such that the difference between the distances from P to the foci F_1 and F_2 is a given constant k

Example: $PF_1 - PF_2 = k$



Hypotenuse - The side of a right triangle opposite the right angle; the longest side of a right triangle

Example:



Hypotenuse and leg triangle congruence - If there exists a one-to-one correspondence between the vertices of two right triangles such that the hypotenuse and leg of one right triangle are congruent to the hypotenuse and corresponding leg of the second right triangle, then the triangles are congruent

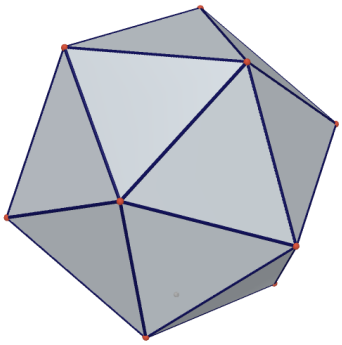
Hypothesis - An assumed statement used as a premise in a proof; the “given”; the “if” clause of an “if-then” statement

I

i - The symbol representing the basic unit of imaginary numbers, $i = \sqrt{-1}$

icosahedron - A polyhedron having twenty faces. A regular icosahedron is one of the five Platonic solids and has twenty equilateral triangles as faces

Example:



Identities - Equations that are true for all values of the variables they contain

Example:

$x + 2 = x + 2$

$Sin^2\theta + Cos^2\theta = 1$

Identity elements - For a binary operation * and a set S, I is the identity element if *a* * I = *a* and I * *a* = *a* for every element *a* that is in S

Examples:

1. For addition (+) and the set of Integers, the number 0 is the identity element, because for every integer *a*: 0 + *a* = *a* and a + 0 = *a*

2. For multiplication (x) and the set of Real Numbers, the number 1 is the identity element, because for every Real Number *a:* 1 x *a* = *a* and *a* x 1 = *a*

Imaginary number - a number of the form *a* + *bi* where *a* and *b* are real numbers and *i* is the square root of -1 ($i = \sqrt{-1}$)

Impossible event/outcome - An event that cannot occur; the probability of an impossible event equals zero.

Incenter of a triangle - The center of the circle that is inscribed in a triangle; the point of concurrence of the three angle bisectors of the triangle which is equidistant from the sides of the triangle

Included angle - The interior angle formed by two sides of a polygon

Included side - The side between two consecutive angles in a polygon

Independent events - Two or more events in which the outcome of one event has no effect on the outcome of any other event

Independent variable - An element in the domain of a function; the input value of a function

Index of a radical - The letter n in the expression $\sqrt[n]{k}$

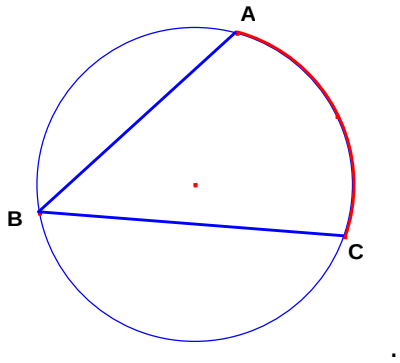
Example: 5 is the index in the expression $\sqrt[5]{32} = 2$

Indirect proof - A method of proof in which the statement that is to be proven is assumed false and a contradiction results

Inductive reasoning - The process of observing data, recognizing patterns and making generalizations about those patterns

Inequality - A mathematical statement containing one of the symbols <, >, $\leq, \geq, \neq$ to indicate the relationship between two quantities

Inscribed angle - An angle whose vertex lies on the circle and whose sides are chords of a circle



Inscribed circle - A circle in the interior of a polygon that is tangent to each side of the polygon

Integers - The set of numbers that is the union of the counting numbers, their opposites, and zero (i.e., {…-4, -3, -2, -1, 0, 1, 2, 3, 4, ... })

Integral - Integration together with differentiation is one of the two main operations in calculus. Given a function *f* of a real variable *x* and an interval [*a*, *b*] of the real number line, the definite integral is defined informally to be the area of the region in the *xy*-plane bounded by the graph of *f*, the *x*-axis, and the vertical lines *x* = *a* and *x* = *b*.

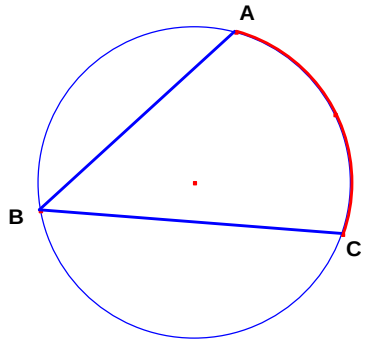
$$\int_a^b f(x)dx$$

The term integral may also refer to the notion of antiderivative, a function $F(x)$ whose derivative is the given function $f(x)$. In this case, it is called an indefinite integral:

$$F(x) = \int f(x)dx$$

Intercepted arc - An arc of a circle whose endpoints lie on the sides of an angle, and all of the points on the arc are in the interior of the angle

Example:



$\angle ABC$ intercepts arc AC

Interior - The set of all points inside a geometric figure

Interpolate - The process of using a given data set to estimate the value of a function or measurement between the values already known; a method of constructing new data points within the range of a discrete set of known data points

Interquartile range - The difference between the first and third quartiles; a measure of variability resistant to outliers

Intersecting lines - Lines that share a common point

Intersection of sets - The intersection of two or more sets is the set of all elements that are common to all of the given sets

Example: If A = {1,2,3,6} and B = {0,2,5,6,7}, then the intersection of A and B, denoted by A ∩ B, is {2,6}

Invariant - unchanged by specified mathematical or physical operations or transformations

Inverse - opposite in order, nature, or effect

Inverse function - If the inverse of a function is also a function it is then an inverse function.

Inverse of a function - The relation formed when the independent variable is exchanged with the dependent variable in a given relation

Inverse of a statement - A statement formed by negating both the hypothesis and conclusion of a given conditional

Example: Given $p \to q$ the inverse is: $\sim p \to \sim q$

Inverse operation - An operation that undoes another operation; addition and subtraction are inverse operations; multiplication and division are inverse operations; raising to a power and taking a root are inverse operations

Inverse trigonometric functions - Given the value of a trigonometric function for an angle θ the inverse trigonometric function outputs the measure of angle θ. The inverses of the six basic trigonometric functions are written as $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$, $\csc^{-1}$, $\sec^{-1}$, and $\cot^{-1}$

Inverse variation - A relationship in which the product of two variables is constant. An inverse variation has an equation in the form $y = \frac{k}{x}$, where *x* and *y* are variables and k is a constant; inversely proportional - proportional to the reciprocal of the independent variable

Interval notation - a (real) interval is a set of real numbers with the property that any number that lies between two numbers in the set is also included in the set; the interval of numbers between *a* and *b*, including *a* and *b*, is often denoted [*a*, *b*]. The two numbers are called the endpoints of the interval.

$$(a,b) = \{x \in \mathbb{R} : a < x < b\}$$
$$[a,b] = \{x \in \mathbb{R} : a \leq x \leq b\}$$
$$[a,b) = \{x \in \mathbb{R} : a \leq x < b\}$$
$$(a,b] = \{x \in \mathbb{R} : a < x \leq b\}$$
$$(a,+\infty) = \{x \in \mathbb{R} : x > a\}$$
$$[a,+\infty) = \{x \in \mathbb{R} : x \geq a\}$$
$$(-\infty,b) = \{x \in \mathbb{R} : x < b\}$$
$$(-\infty,b] = \{x \in \mathbb{R} : x \leq b\}$$
$$(-\infty,+\infty) = \{x \in \mathbb{R}\}$$

Irrational number - A number that cannot be expressed as the ratio of two integers; an irrational number, in decimal form, is non-repeating and non-terminating

Example: The numbers π and $\sqrt{2}$ are irrational numbers

Isometry - A transformation of the plane that preserves distance. If *P'* is the image of *P*, and *Q'* is the image of *Q*, then the distance from *P'* to *Q'* is the same as the distance from *P* to *Q*

Isosceles trapezoid - A trapezoid in which the non-parallel sides are congruent

Isosceles triangle - A triangle that has at least two congruent sides

J

Joint probability function – In statistics, a function that gives the probability that each of two or more random variables takes on a particular value

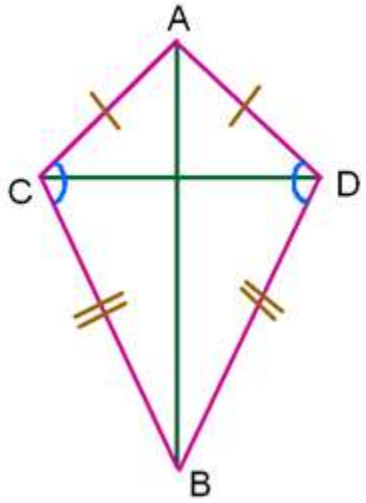
Joint variation – two or more of any kind of variation;

Example: *y* varies directly as the square of *x* and inversely as *z*

$$y = \frac{kx^2}{z}$$

K

Kite – A quadrilateral which has two pairs of adjacent sides equal. The longer diagonal AB divides the kite into two congruent triangles. The shorter diagonal CD divides the kite into two isosceles triangles.



L

Lateral area of a prism - The sum of the areas of the faces of the prism not including the bases

Lateral edge - A line segment that is the intersection of any two lateral faces of a polyhedron

Lateral face - A face of a polyhedron, not including its bases

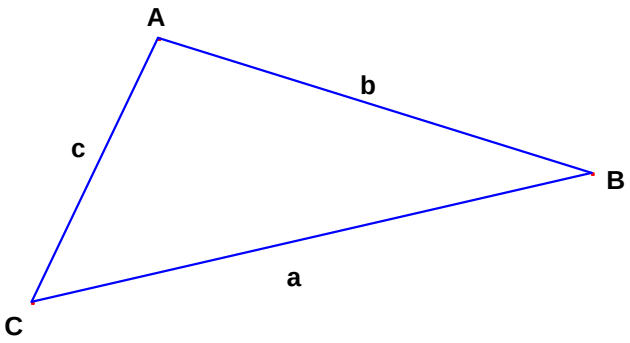
Law of cosines - In any triangle, the square of any side of a triangle is equal to the sum of the squares of the other two sides minus twice the product of the two sides and the cosine of the included angle. In the formulas below, the sides are *a*, *b*, and *c*, and the angles are *A*, *B*, and *C*. The law is used for solving triangles when given two sides and the included angle, or three sides.

Examples for $\triangle$ABC:

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

Law of sine's - In any triangle, the ratio of one side to the sine of its opposite angle is equal to the ratio of any other side to the sine of its opposite angle. The law is used for solving triangles when given one side and two angles or two sides and an angle opposite one of the sides.

Examples for any $\triangle$ABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$



Laws of exponents - Rules involving operations on expressions with like bases

Example: Multiplication law: $x^a \cdot x^b = x^{a+b}$
Division law: $x^a \div x^b = x^{a-b}$, $x \neq 0$
Power law: $\left(x^a\right)^b = x^{ab}$

Laws of logarithms - The rules of logarithmic expressions having like bases

Examples:
Product rule: $\log_b AB = \log_b A + \log_b B$, $b > 0$ and $b \neq 1$

Quotient rule: $\log_b \frac{A}{B} = \log_b A - \log_b B$, $b > 0$ and $b \neq 1$

Power rule: $\log_b A^B = B \log_b A$, $b > 0$ and $b \neq 1$

Leading coefficient - The coefficient of the first term of a polynomial when the polynomial is in standard form

Examples:

5 is the leading coefficient of $5x^2 - 9x + 7$
-4 is the leading coefficient of $1 - 4n^2 + 7n$

Least Common Denominator (LCD) - or lowest common denominator is the least common multiple of the denominators of a set of fractions. It is the smallest positive integer that is a multiple of the denominators. The "cross-multiply" method of comparing fractions effectively creates a common denominator by multiplying both denominators together but the denominator can become large very quickly.

Example: $\frac{5}{12}, \frac{11}{18}$ LCD = 36

Least common multiple (LCM) – the smallest number that is divisible by each member of a set of numbers; e.g. for the set containing 10 and 4 the LCM is 20; to find the LCM of two or more expressions, find the prime factors of each expression; take each factor the greatest number of times it occurs in any one of the expressions and find the product of the different prime factors.

Example:

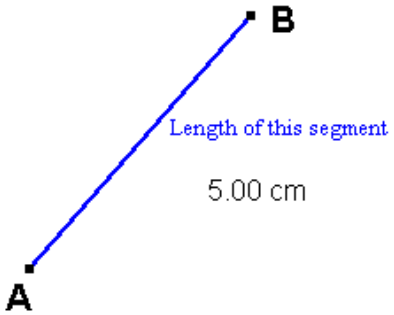
$$12 = 2 \times 2 \times 3$$
$$30 = 2 \times 3 \times 5$$
$$LCM = 2 \times 2 \times 3 \times 5 = 60$$

Least squares regression line - The line that fits data points such that the sum of the squares of the vertical distances between the predicted values on the line and the actual values is minimized

Legs of a right triangle - The two sides of a right triangle that form the right angle

Length of line segment - The distance between the end two end points of a line segment

Example: The length of $\overline{AB}$ is AB



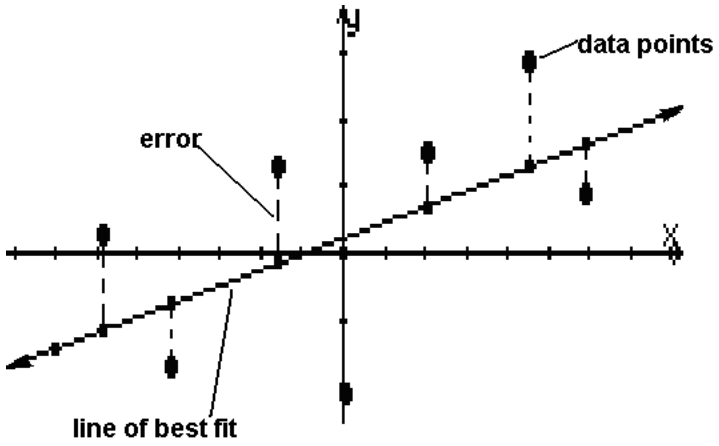
Like radical terms - Terms that have the same index and the same radicand

Examples:
$4\sqrt[3]{12}$ and $-5\sqrt[3]{12}$ are like radical terms
$2\sqrt{5y}$ and $17\sqrt{5x}$ are not like radical terms

Limit of a function - the limit of a function is a fundamental concept in calculus and analysis concerning the behavior of a function near a particular input; the $\lim_{x \to p}(fx) = L$ means that f(x) can be made as close as desired to *L* by making *x* close but not equal to *p*

Line of best fit - A line used to approximate and generalize the linear relationship between the independent and dependent variables for a set of data. It may not be equivalent to a least squares regression model.

Example:



Line segment - Given any two points A and B, $\overline{AB}$ is equal to the union of points A, B, and all of those points between A and B

Line symmetry - A geometric figure has line symmetry if the figure is the image of itself under a reflection in a line.

Linear equation – An equation in which no term is higher than the first degree. Such an equation is called linear because its graph is always a straight line.

Examples:
y = 6x + 8
12n – 19 = 23 + 7n

Linear inequality - An inequality of the first degree

Examples:
4s + 3 > -1
43n + 66 $\leq$ 13 – 17n
y > 9x – 22

Linear pair of angles - Any two adjacent angles whose non-common sides form a line

Linear regression - statistical technique that defines a line that best fits a set of data points and predicts the value of an outcome variable from the values of one or more continuous variables

Linear system - A set of two or more linear equations with common variables

Linear transformation - A transformation of data set X is of the form X' = a + bX, where a is the additive component and b is the multiplicative component

Example: Applying the linear transformation X' = 10 + 2X to the data set X = {3, 5, 6, 8} gives the resulting set X' = {16, 20, 22, 26}

Literal equation - An equation that contains more than one variable

Example: 2a + 3b = c

Locus of points - The set of all points satisfying a given condition or conditions

Example: The locus of points that are equidistant from the endpoints of a line segment is the perpendicular bisector of that line segment.

Logarithm - The exponent, *n*, to which the base *b* must be raised to equal *a,* written as $\log_b a = n$.

Example: $\log_2 8 = 3$ since $2^3 = 8$

Logarithmic form - The expression or an equation containing logarithms

Example: The equation $\log_3 y = x$ is the logarithmic form of the exponential equation $3^x = y$

Logic – the study of the principles which distinguish correct from incorrect reasoning (inference, argument) and the codification of such principles in systems of rules and axioms for the purpose of explicit validation of such reasoning.

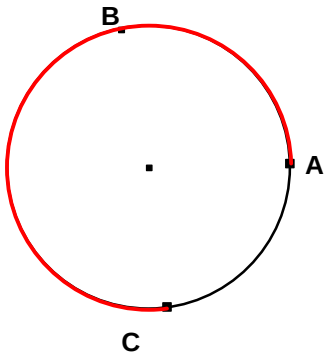
Logical argument - A reasoning process based on logic that uses a series of statements leading to a conclusion

Logical equivalence - Statements that have the same truth value

M

Major arc - In a circle, any arc whose length is greater than the length of a semicircle

Example:



Major arc *ABC*

Matrix – a rectangular array of numbers

Mean - A measure of central tendency denoted by $\bar{x}$, read "x bar", that is calculated by adding the data values and then dividing the sum by the number of values; also known as the arithmetic mean or arithmetic average

Mean proportional - The mean proportional also called the geometric mean of two numbers a and b is the square root of their product. If $\frac{a}{m} = \frac{m}{b}$ then $m = \sqrt{ab}$ is the geometric mean of a and *b.*

Measure of an arc - The measure of the central angle that subtends the arc

Measure of central angle - The measure equal to the degree measure or radian measure of the arc intercepted by the angle

Measure of central tendency - A summary statistic that indicates the typical value or center of an organized data set. The three most common measures of central tendency are the mean, median, and mode.

Measures of dispersion - An indication of the spread, or variation, of data values about the mean. Some common measures of dispersion are range, quartiles, interquartile range, standard deviation and variance.

Median - A measure of central tendency that is, or indicates, the middle of a data set when the data values are arranged in ascending or descending order. If there is no middle number, the median is the average of the two middle numbers.

Examples:

The median of the set of numbers: {2, 4, 5, 6, 7, 10, 13} is 6
The median of the set of numbers: {6, 7, 9, 10, 11, 17} is 9.5

Median of a trapezoid - A line segment that connects the midpoints of the two non-parallel sides of the trapezoid

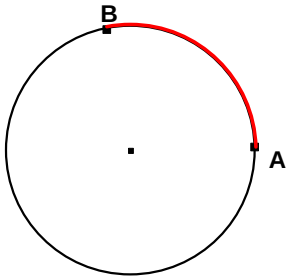
Median of a triangle - A line segment that connects any vertex of a triangle to the midpoint of the opposite side.

Midpoint - A point that divides a line segment into two equal line segments

Midsegment - A line segment that connects the midpoints of two sides of a triangle; also called the midline

Minor arc - In a circle, any arc whose length is less than the length of a semicircle

Example:



Minor arc AC

Minute - A unit of degree measure equal to $\frac{1}{60}$ of a degree

Mode - A measure of central tendency that is given by the data value(s) that occur(s) most frequently in the data set.

Examples:
The mode of the set of numbers {5, 6, 8, 6, 5, 3, 5, 4} is 5
The modes of the set of numbers {4, 6, 7, 4, 3, 7, 9, 1, 10} are 4 and 7
The mode of the set of numbers {0, 5, 7, 12, 15, 3} is none or there is no mode

Monomial - A single term consisting of a product of numbers and variables

Examples: $6, -\frac{3}{4}, x^2, \frac{1}{8}x^5, 8a^2b, -5.9y, m^2n^3p^4$

Multiplication property of zero - For every number a, 0 • a = 0 and a • 0 = 0.

Mutually exclusive events Two events that cannot occur at the same time

N

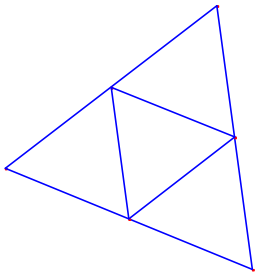
Natural number – a positive integer; also known as the counting numbers, e.g. 1,2,3, …

Negation - For any given statement p, its negation is the statement, $\sim p$ (not p) whose truth value is the opposite of the truth value of p

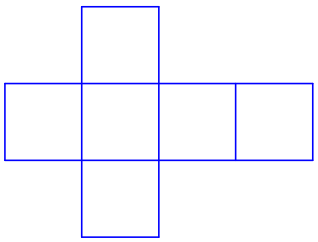
Negative exponent - An exponent that is a negative number. In general, $x^{-a} = \frac{1}{x^a}, x \neq 0$

Net - A two dimensional pattern consisting of polygons which can be folded to form a polyhedron

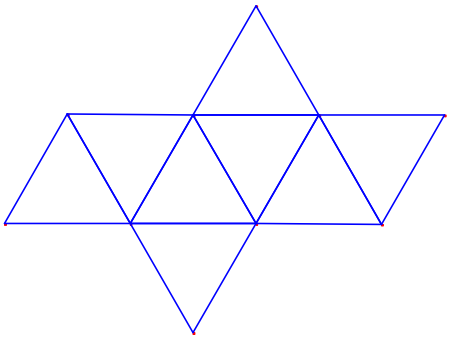
Example: The following are the nets for the Platonic solids.



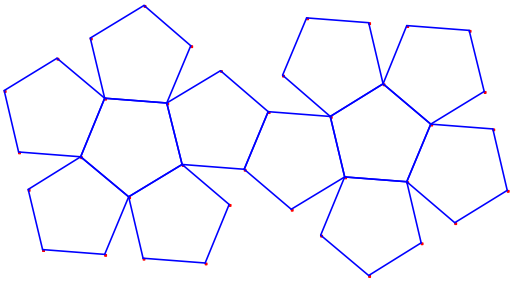
tetrahedron



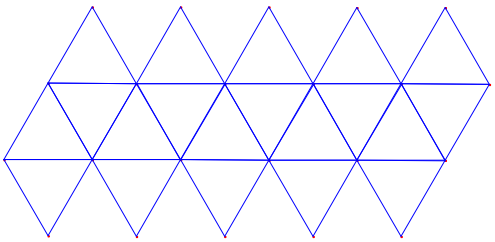
hexahedron (cube)



octahedron



dodecahedron



icosahedron

N-gon - A polygon with n sides

Nonagon - A polygon with nine sides

Non-collinear points - Three or more points that do not lie on the same line

Non- coplanar points - Four or more points that do not lie on the same plane

Non-Euclidean geometry - A geometry that contains an axiom which is equivalent to the negation of the Euclidean parallel postulate

Examples:

Riemannian geometry - A non-Euclidean geometry using as its parallel postulate any statement equivalent to the following: If l is any line and P is any point not on l, then there are no lines through P that are parallel to l. (Also called elliptic geometry)

Hyperbolic geometry - A non-Euclidean geometry using as its parallel postulate any statement equivalent to the following: If l is any line and P is any point not on l, then there exists at least two lines through P that are parallel to l.

Normal curve - The graph of a normal probability density function. This graph is bell-shaped and symmetric about the mean

Normal distribution - A distribution of data that varies about the mean in such a way that the graph of its probability density function is a normal curve; the height of the curve is specified by the mean and standard deviation of the distribution

Nth root - The solution of $x^n = c$ when n is odd or the nonnegative solution of $x^n = c$ when n is even and nonnegative. For any real number c and any positive integer n, the nth root of c is denoted by either $\sqrt[n]{c}$ or $c^{\frac{1}{n}}$

Nth term - The final term of a finite sequence of elements $a_1, a_2, \ldots, a_n,$ or an arbitrary term of an infinite sequence

Null set - The set with no elements; the empty set can be written $\varnothing$ or { }

O

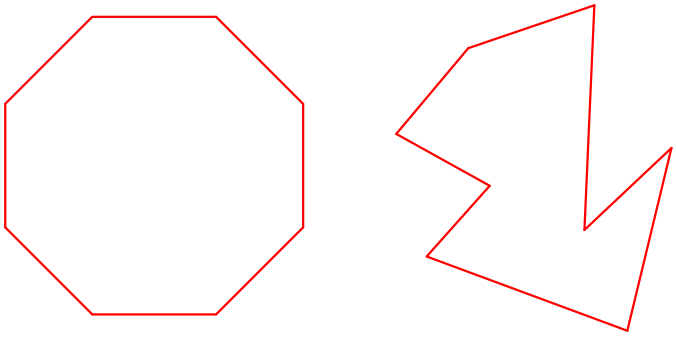
Oblique line and a plane - A line and a plane that are neither parallel nor perpendicular

Obtuse angle - An angle whose measure is greater than 90 degrees and less than 180 degrees

Obtuse triangle - A triangle having one obtuse angle

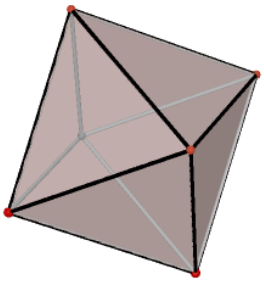
Octagon - A polygon with 8 sides

Examples:



Octahedron - A polyhedron having eight faces. A regular octahedron is one of the five Platonic solids and has eight equilateral triangles as faces

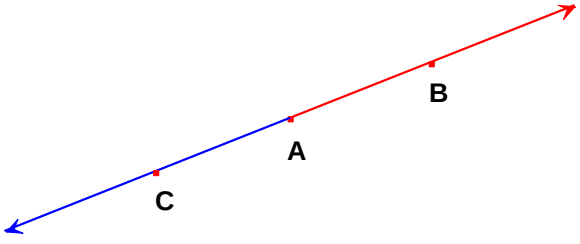
Example:



One-to-one function - A function in which there is only one value of x for every value of y and one value of y for every x ; a function where the inverse is also a function

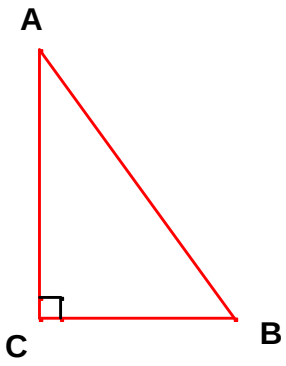
Opposite rays - Two collinear rays whose intersection is exactly one point.

Example: If A is between B and C, then $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are opposite rays.



Opposite side in a right triangle - The side across from an angle. In a right triangle the hypotenuse is opposite the right angle and each leg is opposite one of the acute angles.

Example: With respect to $\angle A$, $\overline{BC}$ is the opposite side, and $\overline{AC}$ is the adjacent side. With respect to side $\overline{AC}$, $\angle B$ is the opposite angle



Order of operations – The sequence in which the various operations in a mathematical expression are to be evaluated; the indicated operations of parentheses and/or brackets, exponents or powers, and roots are performed first followed by multiplication and division in the given order, followed by addition and subtraction; the acronym PEMDAS or "Please Excuse My Dear Aunt Sally" is often used as a mnemonic; It stands for Parentheses, Exponentiation, Multiplication, Division, Addition, Subtraction

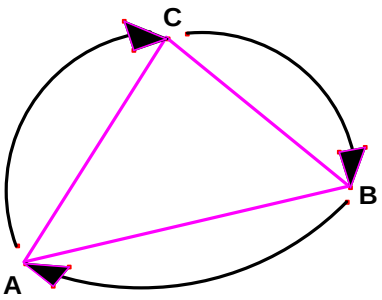
Ordered pair - Two numbers that are used to identify the position of a point in a plane; the two numbers are called coordinates and are represented by (x, y)

Ordered triple - Three numbers that are used to identify the position of a point in space; the three numbers are called coordinates and are represented by (x, y, z)

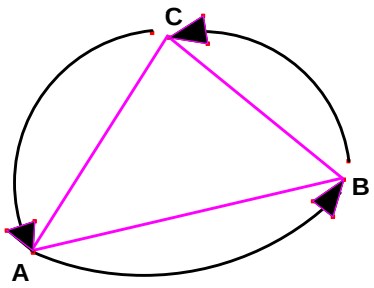
Ordinate - The vertical coordinate of a two-dimensional rectangular coordinate system; usually denoted by y

Orientation - The arrangement of the points, relative to one another, after a transformation; the reference made to the direction traversed (clockwise or counterclockwise) when traveling around a geometric figure.

Example:



ΔACB has a clockwise orientation



ΔABC has a counterclockwise orientation

Origin - The point in the Cartesian coordinate plane at which the horizontal and vertical axes intersect, designated by the ordered pair (0, 0)

Orthocenter - The point of concurrence of the three altitudes of a triangle

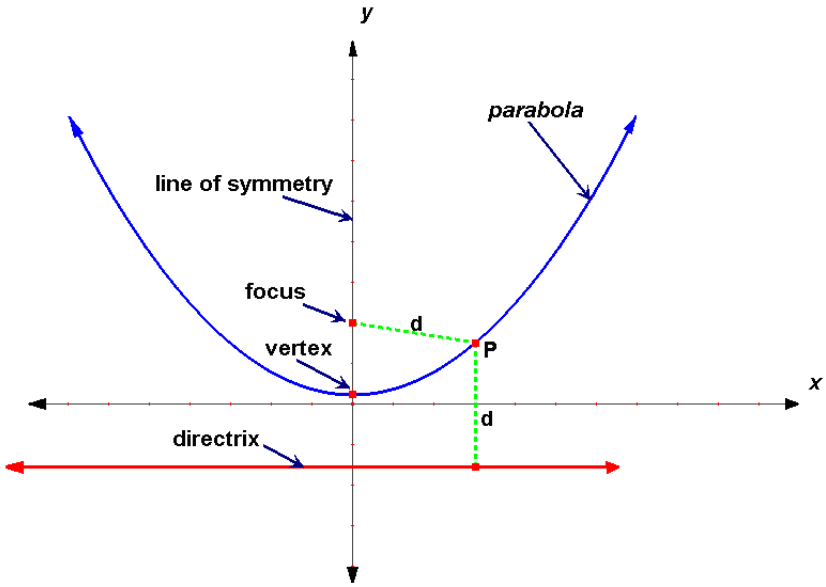
Outlier - A data value that is far removed from the body of the data

> **Example:** Given the data set {2, 4, 5, 16, 22, 112}, 112 is the outlier. The value of the outlier will greatly effect on the value of the mean but not the median.

P

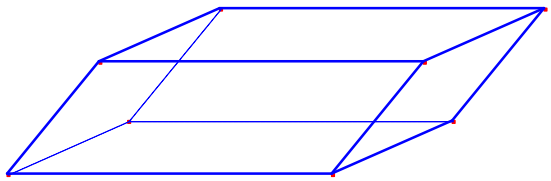
Parabola - The locus of points equidistant from a given point (called the focus) and a given line (called the directrix). A common form of an equation of a parabola with vertical line symmetry is $y = ax^2 + bx + c$, where a, b, and c are real numbers and $a \neq 0$. A parabola is the set of all points in a plane whose distance from a fixed point is equal to its distance from a fixed line. The fixed point is called the focus and the fixed line is the directrix. The line passing through the focus and perpendicular to the directrix is called its axis of symmetry, and the point where the parabola intersects its axis of symmetry is called its vertex. The line segment that joins two points on the parabola, passes through the focus, and is perpendicular to the axis of symmetry is called the focal width. The vertex is midway between the focus and the directrix.

Example:



Parallelepiped -A prism whose bases are parallelograms

Example:



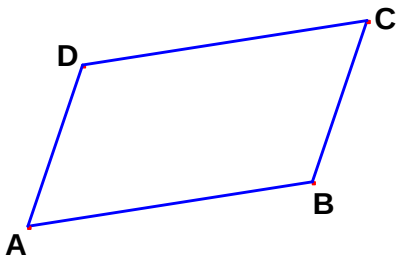
Parallel lines - Two or more coplanar lines that do not intersect; parallel line segments or rays are line segments or rays that are subsets of parallel lines

Parallel planes - Two or more planes that do not intersect

Parallel postulate - also called Euclid's fifth postulate because it is the fifth postulate in Euclid's Elements. It states that If a line segment intersects two straight lines forming two interior angles on the same side that sum to less than two right angles, then the two lines, if extended indefinitely, meet on that side on which the angles sum to less than two right angles.

Parallelogram - A quadrilateral in which both pairs of opposite sides are parallel

Example:



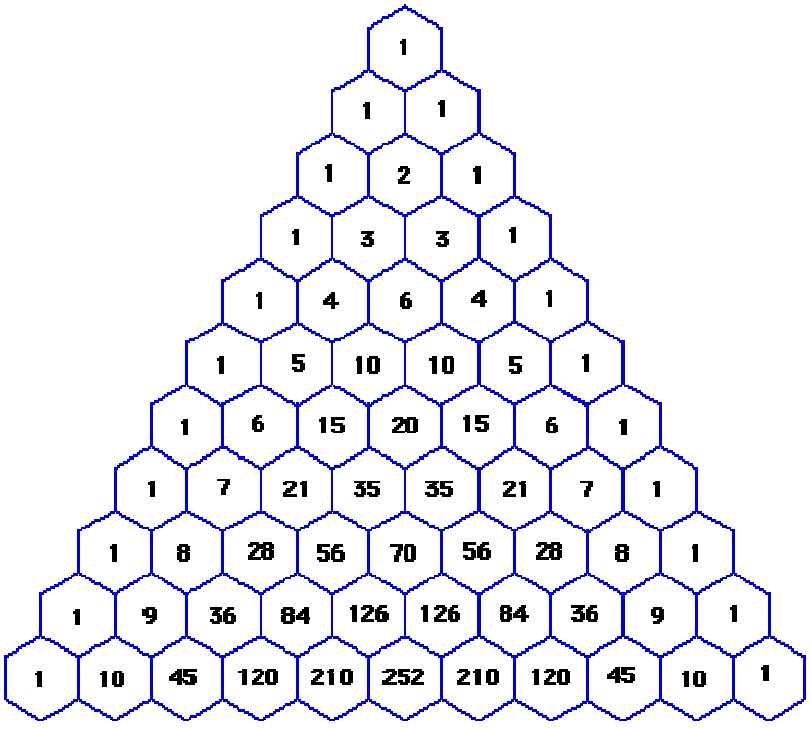
$\overline{AB} \parallel \overline{CD}$ and $\overline{AD} \parallel \overline{BC}$

Parameter - A quantity or constant whose value varies with the circumstances of its application

Example: In $y = ax^2$ *a* is a parameter

Partial fraction – A procedure used to reduce the degree of either the numerator or the denominator of a rational function

Pascal's Triangle - A triangular array of numbers composed of the coefficients of $(x+y)^n$ where *n* is any whole number. Each row begins and ends with 1. The other values are found by adding the two numbers that are above and on either side of that value

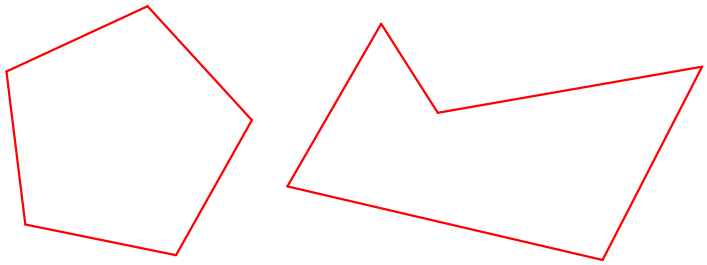


Example: The coefficients of $(x+y)^4$ are the numbers in the 4th row of Pascal's Triangle

$$(x+y)^4 = 1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4$$

Pentagon - A polygon with 5 sides

Examples:



Percent - a part of a whole expressed in hundredths; the term for expressing a fraction having a denominator of one hundred or in terms of an equivalent decimal; for example, three percent means three out of every hundred or three hundredths of a given number. In computations, percent is called the rate, as in the rate of interest.

Percentage - the result obtained by taking a given percent (multiplying a number by a percent), or rate, of a given quantity called the base. The percentage equals the rate times the base.

Percentile - A score below which a certain percentage of the scores in a distribution fall.

> **Example:** If a test score of 87 is the 75th percentile of a distribution, then 75% of the scores are less than 87 and 25% of the scores are greater than or equal to 87.

Percent of increase/decrease - The magnitude of increase/decrease expressed as a percent of the original quantity

> **Example:** Mr. Smith received a raise of $5,000 on his annual salary: His previous salary was $25,000, and his new salary is $30,000. The raise is a 20% increase of his salary ($5,000 is 20% of $25,000).

Perimeter - The sum of the lengths of all the sides of a polygon

Period (of a function) - The horizontal distance after which the graph of a function starts repeating itself; the smallest value of k in a function f for which there exists some constant k such that $f(t) = f(t+k)$ for every number t in the domain of f

Periodic function - An oscillating function that repeats its values at regular intervals; a function f for which there exists some constant k such that $f(t) = f(t+k)$ for every number t in the domain of f

Permutation - An arrangement of objects in a specific order

Perpendicular bisector - A line, segment or ray that is perpendicular to a line segment at its midpoint.

Perpendicular lines - Two lines that intersect to form right angles

Perpendicular planes - Two planes that intersect to form right dihedral angles.

Phase shift - The horizontal translation of a periodic graph

Pi - the ratio of the circumference to the diameter of a circle; approximately equal to 3.14159265358979323846

Place value – the position of a digit in a number to denote its value; in base 10, the first place from the right is the unit's digit, the second place is the tens digit, the third digit is the hundreds digit and so on

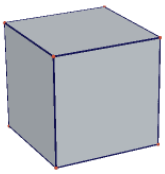
Plane – A surface in which a straight line joining any two of its points will lie wholly in the surface. Planes have the following properties:

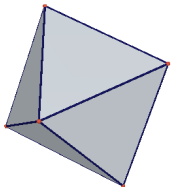
1. A plane is determined by three noncollinear points
2. A plane is determined by a line and a point not on the line
3. A plane is determined by two intersecting lines
4. A plane is determined by two parallel lines
5. The intersection of a plane and a straight line not in the plane is a point
6. The intersection of two planes is a straight line
7. An infinite number of planes can pass through one line or point

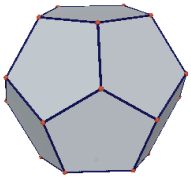
Platonic solids - The five regular polyhedral: tetrahedron, cube, octahedron, dodecahedron and icosahedron

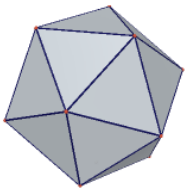
Example:


Tetrahedron


Cube


Octahedron


Dodecahedron


Icosahedron

Point - An undefined term in geometry usually visualized as a dot representing a non-dimensional location in space

Point of concurrency - A point that is the intersection of three or more lines

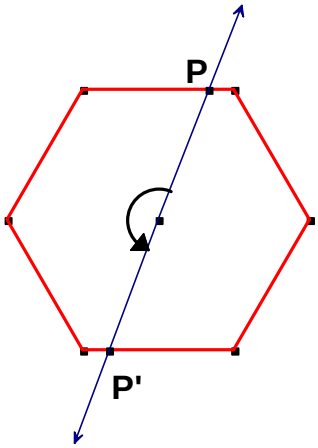
Point of tangency - The point where a tangent line intersects a curve

Point-slope equation of a line - The equation of a line formed using its slope and the coordinates of a point on the line, where *m* is the slope of the line and (x_1, y_1) are the coordinates of the given point

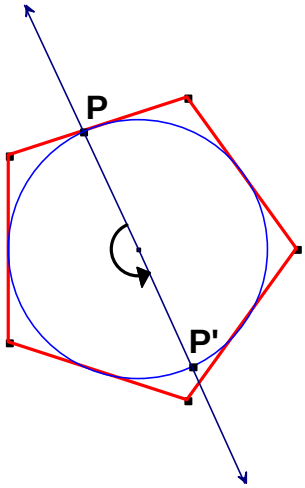
Example: If the coordinates of a point on the line are (x_1, y_1) and the slope is m, then the equation of the line is $(y - y_1) = m(x - x_1)$

Point symmetry - A geometric figure has point symmetry if every point on the figure is the image of itself under a rotation of 180° about some fixed point

Examples:

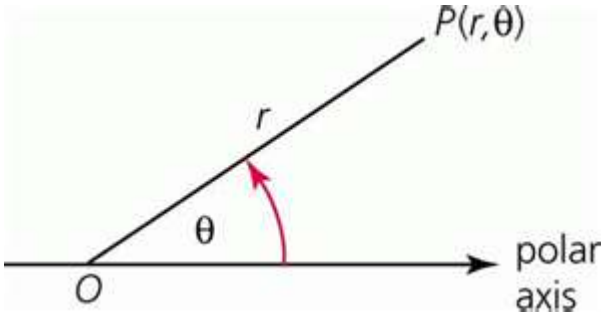


A regular hexagon has point symmetry about its center



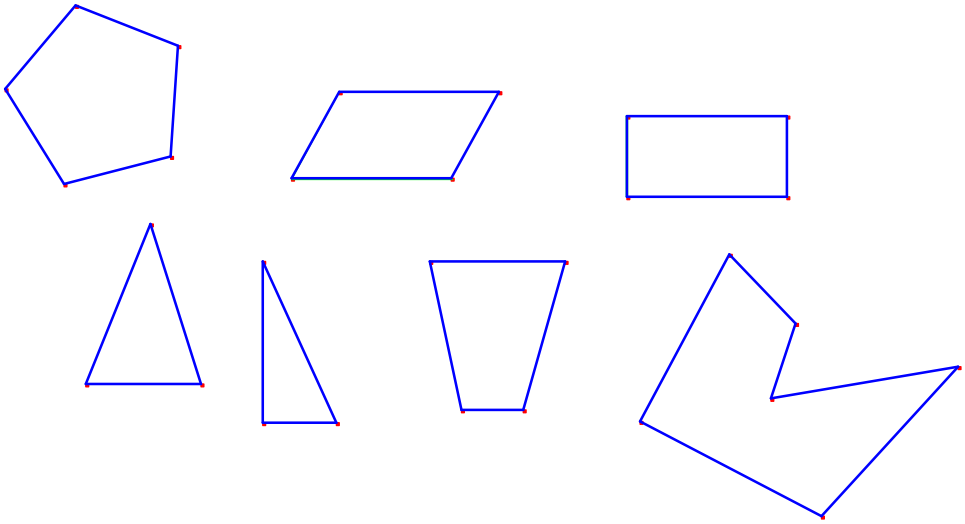
A pentagon does not have point symmetry

Polar coordinates - a two-dimensional coordinate system in which each point on a plane is determined by a distance from a fixed point and an angle from a fixed direction. The ray is called the polar axis and the end-point is called the pole or origin. The segment joining the point to the pole is the radius vector, denoted by *r*. Polar coordinates of *P* are the values *r* and θ written as an ordered pair (*r*, θ) such that *r* = *OP* and θ is the radian measure of any angle whose initial side is the fixed ray and whose terminal side contains P.



Polygon - A closed plane figure formed by three or more line segments that meet only at their endpoints

Examples:



Polyhedron - A solid figure bounded by polygons

Polynomial - An algebraic expression having more than one term

Polynomial expression - A polynomial expression in terms of x that can be written in the form $a_n x^n + a_{n-1} x^{n-1} + ... + a_2 x^2 + a_1 x + a_0$ where n is a nonnegative integer and a_i is a real number

Polynomial function - A function that can be written in the form $f(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_2 x^2 + a_1 x + a_0$, where n is a nonnegative integer and a_i is a real number

Position vector - A coordinate vector whose initial point is the origin. Any vector can be expressed as an equivalent position vector by translating the vector so that it originates at the origin.

Postulate - A statement assumed to be true without proof

Powers of *i* - The repetitive pattern of imaginary numbers; the following pattern repeats every fourth time:

$i^0 = 1$
$i^1 = i$
$i^2 = -1$
$i^3 = -i$
$i^4 = 1$

Premise - A proposition upon which an argument is based or from which a conclusion is drawn

Prime factorization - Writing an integer as a product of powers of prime numbers

Example: 30 = 2 x 3 x 5

Principal square root - The positive square root of a number

Example: The principal square root of 4 is 2.

Prism - A polyhedron with two congruent, parallel, polygonal bases and whose lateral faces are parallelograms

Probability - The likelihood of an event occurring; the probability of an event must be greater than or equal to 0 and less than or equal to 1

Product property of proportions - In a proportion $\frac{a}{b} = \frac{c}{d}$, the product of the means (*b* and *c*) equals the product of the extremes (*a* and *d*), or in other words: $b \cdot c = a \cdot d$

Proof - A logical argument that establishes the truth of a statement; a valid argument, expressed in written form, justified by axioms, definitions, and theorems

Proof by contradiction - A method of proof which demonstrates the truth of an implication by proving that the negation of the conclusion of that implication leads to a contradiction; also called an indirect proof

Properties of the real numbers - Rules that apply to the operations with real numbers

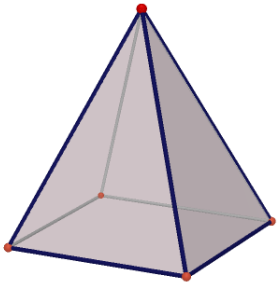
Examples:

Commutative Property	$a + b = b + a$	$ab = ba$
Associative Property	$a + (b + c) = (a + b) + c$	$a(bc) = (ab)c$
Distributive Property	$a(b + c) = ab + ac$	
Identity	$a + 0 = a$	$a \cdot 1 = a$
Inverse	$a + (-a) = 0$	$a \cdot \frac{1}{a} = 1$
Zero Property		$a \cdot 0 = 0$

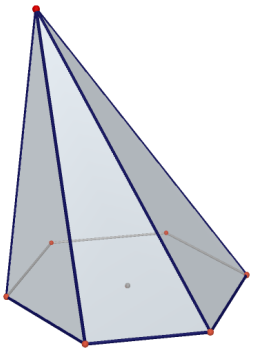
Proportion – the relation between things (or parts of things) with respect to their comparative quantity, magnitude, or degree; a statement of equality between two ratios. The four quantities are called the terms of the proportion. In any proportion the first and fourth terms are called the extremes and the second and third terms are called the means. To solve a proportion, set the two ratios equal and cross multiply. A proportion such as $\frac{a}{b} = \frac{b}{c}$ in which the means are equal is called a mean proportion.

Pyramid - A polyhedron having a polygonal base and triangles as lateral faces.

Example:



right square pyramid



hexagonal pyramid

Pythagorean identities - The trigonometric identities based on the Pythagorean Theorem; the identity and the identities derived from it

Examples:

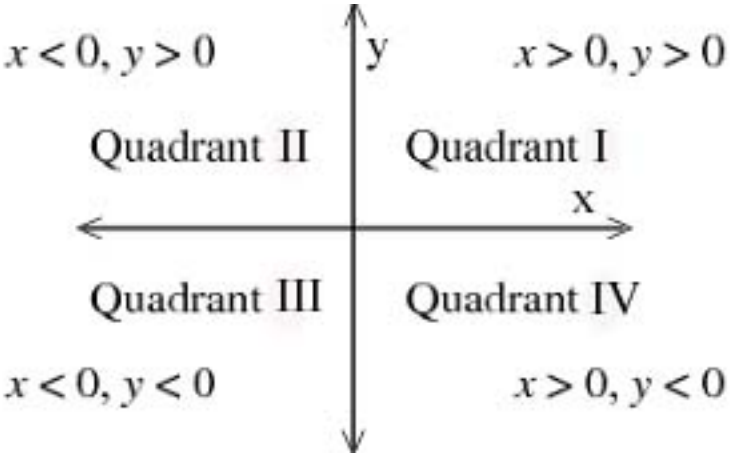
$\sin^2 x + \cos^2 = 1$
$1 + \tan^2 x = \sec^2 x$
$1 + \cot^2 x = \csc^2 x$

Pythagorean theorem - The mathematical relationship stating that in any right triangle the sum of the squares of the lengths of the two legs is equal to the square of the length of the hypotenuse; if *a* and *b* are the lengths of the legs and *c* is the length of the hypotenuse, then $c^2 = a^2 + b^2$

Q

Quadrant - The four regions of a plane created by the intersection of the coordinate axes. Each of these quadrants has a number designation:

Example:



Quadrantal angle - An angle in standard position whose terminal side falls on an axis

Example: $0°, 90°, 180°, 270°, 360°$

Quadratic equation - A polynomial equation of the second degree; an equation that can be written in the form $ax^2 + bx + c = 0$, where *a, b*, and *c* are real constants and $a \neq 0$

Quadratic formula - The formula used to determine the roots (solutions) of a quadratic equation:

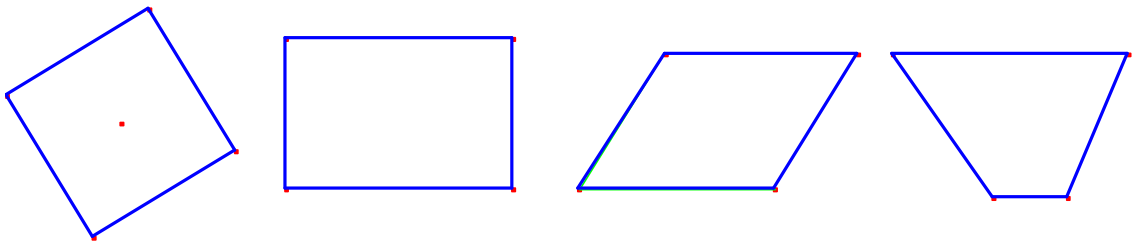
$$ax^2 + bx + c = 0; \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Quadratic inequality - A second degree inequality

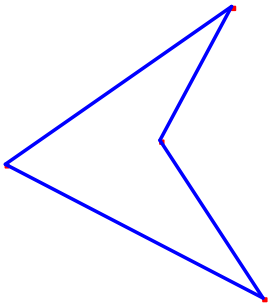
Example: $2x^2 - 5x - 3 \geq 0$

Quadrilateral - A polygon with 4 sides

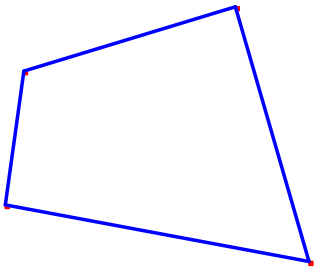
Examples



square rectangle parallelogram trapezoid



concave quadrilateral



quadrilateral

Quantitative - expressible as a quantity or relating to or susceptible of measurement; descriptions using numerical measures such as quantity, height, or age

Quartiles - Values that divide an ordered data set into fourths. The median, or second quartile Q_2, divides the data into a lower half and an upper half; the first quartile Q_1 is the median of the lower half; and the third quartile Q_3 is the median of the upper half

R

Radian - An angle measure in which one full rotation is 2π radians. One radian is the measure of an arc or the measure of the central angle than intercepts that arc such that the arc's length is the same as the radius of that circle; one radian is equal to $\frac{180}{\pi}$ or $\frac{360}{2\pi}$ degrees, or about 57.2958 degrees

Example: π radians = 180 degrees

Radian measure - For an angle in standard position, the length of the arc along the unit circle from the point (1, 0) on the initial side to the point *P* where the terminal side intersects the unit circle.

Radical – an expression used to indicate the root of a number

Radical equation - An equation that contains at least one term under a radical sign

Radical expression - An expression that contains at least one term under a radical sign

Radical form - The use of a radical sign to express a number with a fractional exponent.

Example: $x^{\frac{2}{3}} = \sqrt[3]{x^2}$

Radicand - The quantity under a radical sign; a number or expression from which a root is extracted

Example: 3 is the radicand of $\sqrt{3}$

Example: In the expression $\sqrt[n]{k}$, the radicand is *k*

Radius - A line segment drawn from the center of a circle to a point on the circle

Random sample - A sample in which all members of a population have an equal chance of being selected

Range (of a data set) - The difference between the maximum and minimum data values in a data set

Example: Given the data: 2, 7, -3, 14, –1, 6, 34, 3
The minimum value is -3
The maximum value is 34
The range is 34 – (–3) = 37

Range (of a function) - The set of values of the dependent variable of a given function; the set of second coordinates (generally the *y* variable) in the ordered pairs of a function

Rate - a quantity, amount, or degree of something measured per unit of something else; a ratio between two measurements, often with different units (e.g., miles per hour, price per pound, students per class, and heartbeats per minute)

Ratio - the relationship in quantity, amount, or size between two or more things; the relationship between two quantities of the same kind, expressed as a fraction (e.g., 2 to 3, 2:3, $\frac{2}{3}$)

Rational coefficient - A coefficient that is a rational number

Rational equation - An equation that contains at least one rational expression

Rational expression - The quotient of two polynomials in the form $\frac{A}{B}$, where A and B are polynomials and $B \neq 0$

Example: $\frac{2x+1}{3x^2-9},\ 3x^2-9 \neq 0$

Rational inequality - An inequality which contains a rational expression

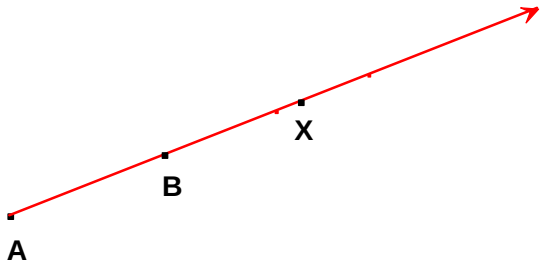
Example: $\frac{2x}{x+4} \leq 2x-5,\ x+4 \neq 0$

Rational number - Any number that can be expressed as a ratio in the form $\frac{a}{b}$ where *a* and *b* are integers and $b \neq 0$; a rational number is either a terminating or repeating decimal

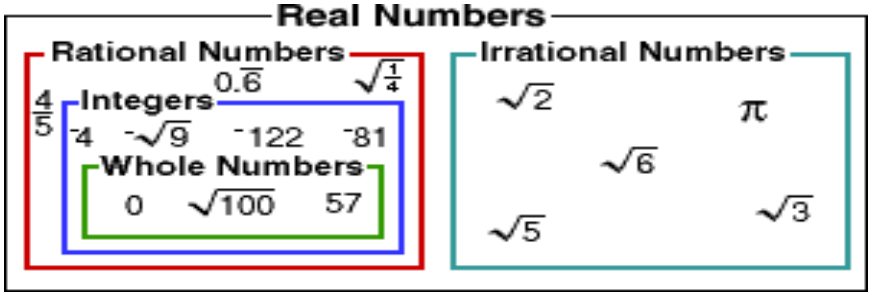
Rationalize a denominator - The process of changing the irrational denominator of a fraction to a rational expression.

Example: $\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$ or $\frac{2+3i}{1-i} \cdot \frac{1+i}{1+i} = \frac{-1+5i}{2}$

Ray - Given any two points A and B, $\overrightarrow{AB}$ is equal to the union of $\overline{AB}$ and all of those points X such that B is between X and A



Real numbers - The set of numbers that includes all rational and irrational numbers



Reason - the capacity for rational thought; the power of comprehending, inferring, or thinking especially in orderly rational ways

Reciprocal – the reciprocal of any number *a* is $\frac{1}{a}$. The product of any number and its reciprocal is 1

Reciprocal trigonometric functions - The six functions:

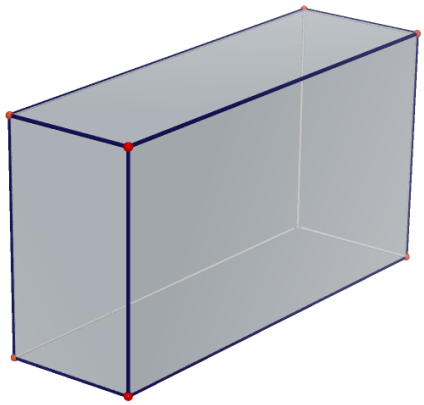
$$\sin x = \frac{1}{\csc x}$$
$$\cos x = \frac{1}{\sec x}$$
$$\tan x = \frac{1}{\cot x}$$
$$\cot x = \frac{1}{\tan x}$$
$$\sec x = \frac{1}{\cos x}$$
$$\csc x = \frac{1}{\sin x}$$

Rectangle - A parallelogram containing one right angle; a quadrilateral with four right angles

Rectangular coordinates - An ordered pair of real numbers that establishes the location of a point in a coordinate plane using the distances from two perpendicular intersecting lines called the coordinate axes. (See also Cartesian coordinates)

Rectangular solid - A prism whose six faces are rectangles

Example:



Recursive rule - For a sequence $a_1, a_2, a_3, ...a_n...$, a formula that requires the computation of all previous terms in order to find the value of a_n

Example:

$a_1 = 2$
$a_2 = 5$
$a_3 = 11$
...
$a_n = 2a_{n-1} + 1$

Reference angle The positive acute angle formed by the x-axis and the terminal side of an angle θ in standard position

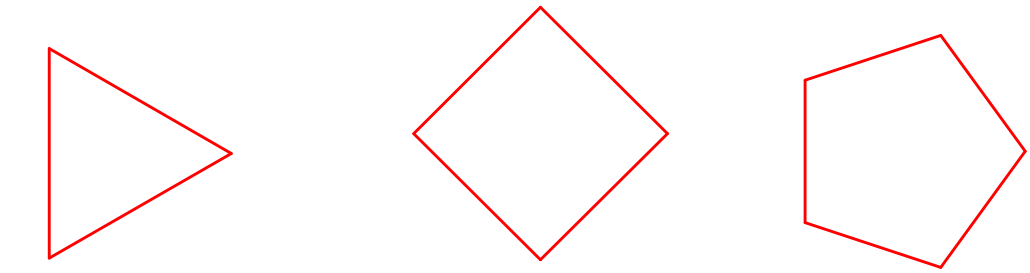
Reflection - An isometry where if l is any line and P is any point not on l, then $r_l(P) = P'$ where l is the perpendicular bisector of $\overline{PP'}$ and if $P \in l$ then $r_l(P) = P$

Reflexive property of equality - A property of real numbers that states $a = a$

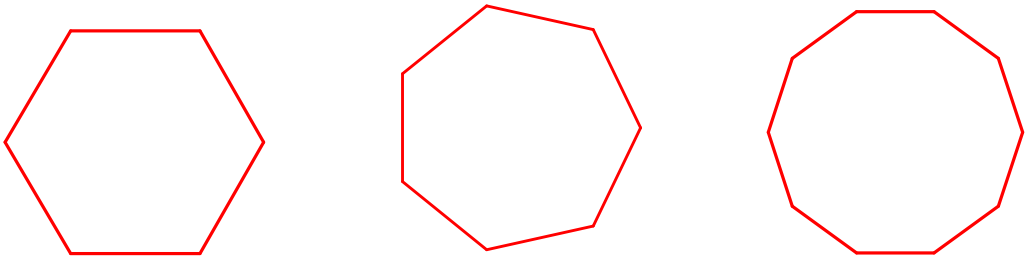
Regression model - A function (e.g., linear, exponential, power, logarithmic) that fits a set of paired data. The model may enable other values of the dependent variable to be predicted.

Regular polygon - A polygon which is both equilateral and equiangular

Example:



equilateral triangle square pentagon



hexagon heptagon decagon

Regular pyramid - A pyramid whose base is a regular polygon and whose lateral faces are congruent isosceles triangles

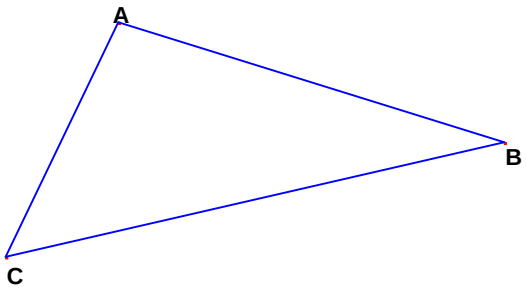
Relation – A property that is true for two objects in a particular order; a correspondence between two sets; a set of ordered pairs

Example:

{(*P*,*Q*)} = {(4,5), (2,20), (7,5), (-4,6)}

Relative error - The ratio of the absolute error in a measurement to the size of the measurement; often written as a percent and called the percent of error; the absolute error is the difference between an approximation and the exact value.

Remote interior angles - Either interior angle of a triangle that is not adjacent to a given exterior angle of the triangle; also called non-adjacent interior angles



With respect to vertex A, $\angle B$ and $\angle C$ are remote interior angles.

Replacement set - The set whose members can be substituted for the variable(s) in an open sentence

Representations - Models, (e.g., symbolic, verbal, graphical, numerical, physical, pictorial) used to represent and interpret mathematical problems

Restricted domain - The domain resulting from a restriction placed on a function, based on the context of the problem

Resultant - The vector that is produced from the addition of two or more other vectors

Rhombus - A parallelogram with two adjacent congruent sides; a quadrilateral with four congruent sides

Right angle – equal adjacent angles formed by two perpendicular lines, the measure of which is 90° or $\frac{\pi}{2}$

Right circular cylinder - A cylinder whose bases are circles and whose altitude passes through the center of both bases

Right circular cone - A cone whose base is a circle and whose altitude passes through the center of its base

Right pyramid - A pyramid whose lateral faces are isosceles triangles

Right triangle - A triangle with one right angle

Root of an equation - A solution to an equation of the form *f*(*x*) = 0

Examples:

A root of the equation $y = 6x - 18$ is 3 because when 3 is substituted in for x, the value of $y = 0$

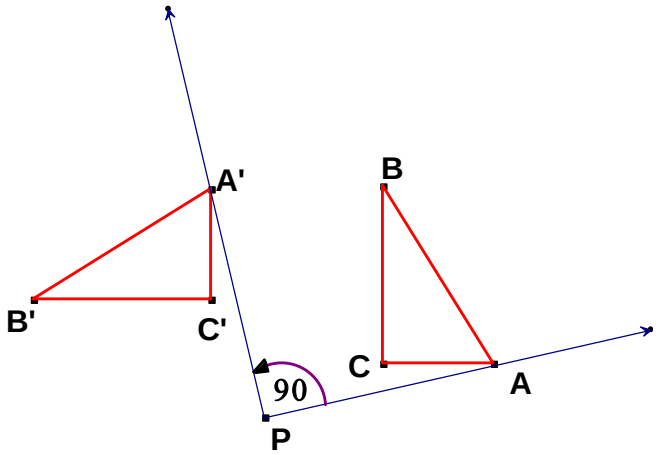
The roots of $x^2 - x - 2 = 0$ are $x = 2$ and $x = -1$. The equation is true if we substitute either $x = 2$ or $x = -1$ into the equation

Roster form - A notation for listing all the elements in a set using set brackets and a comma between each element

Example: The set of prime numbers less than 10, expressed in roster form is {2, 3, 5, 7}

Rotation - An isometry where if P is a fixed point in the plane, θ is any angle and $A \neq P$ then $R_{P,\theta}(A) = A'$ where $m\angle APA' = \theta$ and $R_{P,\theta}(P) = P$

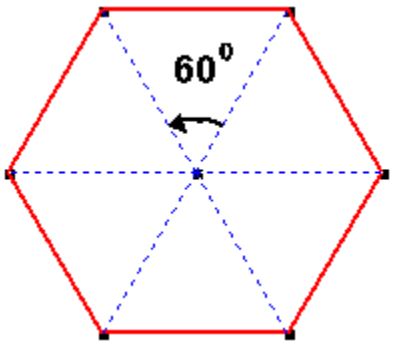
Example:



$R_{P,\,90^\circ}(\Delta ABC) = \Delta A'B'C'$

Rotational symmetry - A geometric figure has rotational symmetry if the figure is the image of itself under a rotation about a point through any angle whose measure is not a multiple of 360°.

Example:



A regular hexagon has rotational symmetry of 60°, 120°, 180°, 240°, and 300°

S

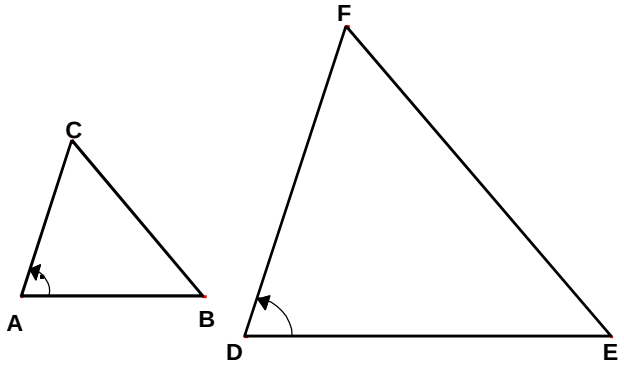
Sample space - The set of all possible outcomes for a given event

Example: The sample space for tossing two coins is:
{(H,H), (H,T), (T,H), (T,T)}.

SAS triangle congruence (Side/Angle/Side) - If there exists a one-to-one correspondence between the vertices of two triangles, such that two sides and the included angle of one triangle are congruent to the corresponding two sides and included angle of the second triangle, then the two triangles are congruent.

SAS Similarity Theorem - If there exists a one-to-one correspondence between the vertices of two triangles, such that two pairs of corresponding sides are proportional and their included angles are congruent, then the two triangles are similar.

Example:



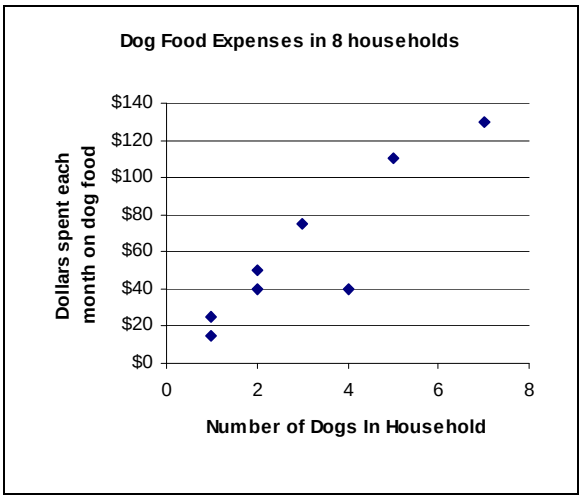
ΔABC is similar to ΔDEF because $\angle BAC \cong \angle EDF$ and $\frac{AC}{AB} = \frac{DF}{DE}$

Scalene triangle - A triangle with no two sides of equal length

Scatter plot - A graphical display of statistical data plotted as points on a coordinate plane to show the correlation between two quantities

Example: Below is a table of paired quantitative data and its scatter plot:

Household	Number of dogs	Dollars spent each month on dog food
Perez	3	75
Jones	2	40
Balcovich	5	110
Parson	1	15
Montego	1	25
Schwartz	7	130
Barton	2	50
Walker	4	40



Scientific notation - A convenient way to write very small or large numbers; in scientific notation, numbers are separated into two parts, a real number with an absolute value equal to or greater than 1 and less than 10 and an order of magnitude value written as a power of 10

Examples:

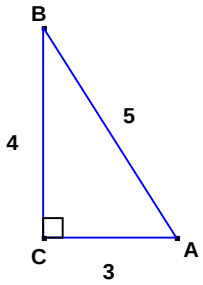
An electron's mass is approximately $0.00000000000000000000000000009109\ \text{Kg}$, which when written in scientific notation is 9.109×10^{-31} Kg.

The earth's mass is approximately $5,973,000,000,000,000,000,000,000\ \text{Kg}$, which when written in scientific notation is $5.973\times10^{24}\ \text{Kg}$

Secant (of a circle) - A line that intersects a circle in exactly two points

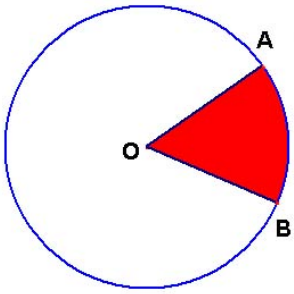
Secant (of an angle) - For a given acute angle θ in a right triangle, $\sec\theta$, is the ratio of the length of the hypotenuse to the length of the side adjacent to the acute angle θ; the reciprocal of the cosine ratio of the given angle. See also circular function.

Example: In this right triangle, $\sec A=\frac{5}{3}$ and $\sec B=\frac{5}{4}$



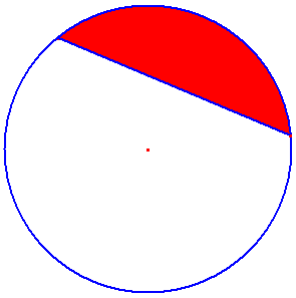
Sector of a circle - A region bounded by an arc of the circle and the two radii to the endpoints of the arc

Example: The shaded area in the circle below is a sector of circle *O*



Segment of a circle - The region bounded by a chord and the arc subtended by that chord

Example: The shaded part of the circle is called a segment of a circle.



Semi-circle - Either of the arcs of a circle determined by the endpoints of a diameter

Set - an abstract collection of elements, numbers or symbols

Set-builder notation - A notation used to describe the elements of a set

Example:

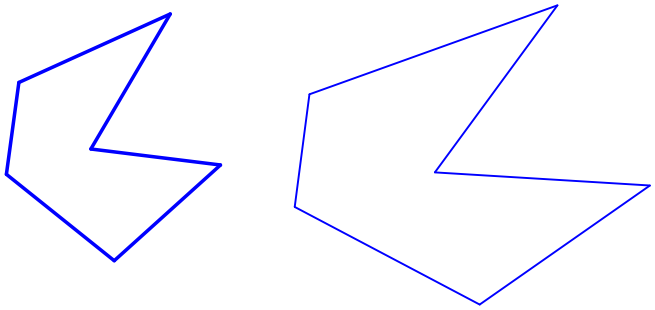
The set of all positive real numbers in set builder notation is $\{x: x \in \mathbb{R} \; and \; x > 0\}$ this is read as "the set of all values of *x* such that *x* is a real number and x is greater than 0."

Sigma notation ($\sum$) - A shorthand way of writing a sum by using the Greek letter sigma

Example: $3+6+9+...+99=\sum_{n=1}^{33} 3n$

Similar polygons - Two polygons which have the same shape but not necessarily the same size

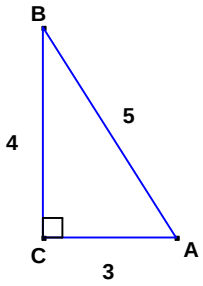
Example:



Simplest form - An expression that has been rewritten as simply as possible using the rules of arithmetic and algebra

Sine - For a given acute angle θ in a right triangle, $\sin\theta$, is the ratio of the length of the side opposite the acute angle θ to the length of the hypotenuse

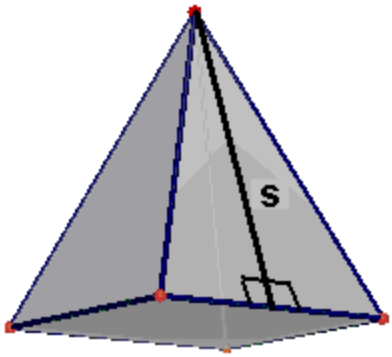
Example: In this right triangle $\sin A = \frac{4}{5}$ and $\sin B = \frac{3}{5}$



Skew lines - Two non-coplanar lines that do not intersect

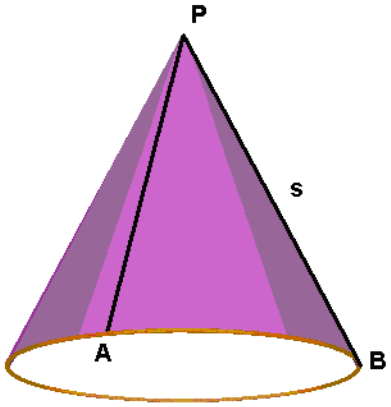
Slant height - Of a *pyramid*: The altitude of a lateral face of a pyramid.

Examples:



S is the slant height of the pyramid

Of a *cone*: The length of a line segment drawn on the lateral surface of a cone from its vertex to a point on the circle that determines its base



$PA = PB = s$ is the slant height of the cone.

Slope - The measure of the steepness of a line; the ratio of vertical change to horizontal change (rise-over-run); if point P is (x_1,y_1) and point Q is (x_2,y_2) the slope of $\overline{PQ}$ is $\frac{\Delta y}{\Delta x}=\frac{y_2-y_1}{x_2-x_1}$

Example:

The slope of the line containing the points A(-3,7) and B(5, -2) is $\frac{(-2)-(7)}{(5)-(-3)}=\frac{-9}{8}$

Slope - intercept equation of a line - The equation of a line formed using its slope and its y-intercept. If the coordinates of the y-intercept of the line are $(0,b)$ and the slope is m, then the equation of the line is $y=mx+b$

Solution set - Any and all value(s) of the variable(s) that satisfy an equation, inequality, system of equations, or system of inequalities

Sphere - The locus of points in space at a given distance from a fixed point

Square - a plane rectangle with four equal sides and four right angles; a four-sided regular polygon

Square units - The basic unit of area

SSS triangle congruence (Side/Side/Side) - If there exists a one-to-one correspondence between the vertices of two triangles, such that all three sides of one triangle are congruent to the corresponding sides of the second triangle, then the two triangles are congruent.

Standard deviation (population) - A measure of variability; standard deviation measures the average distance of a data element from the mean; if data is taken from the entire population, divide by n when averaging the squared deviations. The following is the formula for population standard deviation:

$$\sigma=\sqrt{\frac{\sum(x_i-\overline{x})^2}{n}}$$

Standard deviation (sample) - A measure of variability; standard deviation measures the average distance of a data element from the mean; if data is taken from a sample instead of the entire population, divide by $n-1$ when averaging the squared deviations. The following is the formula for sample standard deviation:

$$s=\sqrt{\frac{\sum(x_i-\overline{x})^2}{n-1}}$$

Standard position (of an angle) - An angle in the coordinate plane with its vertex at the origin and its initial side on the positive x-axis

Straightedge - An object with no marked units of measure that is used for drawing straight lines

Subscript – A small number or letter written below and to the right or left of a letter as a mark of distinction or as part of an operative symbol. Used on a variable to denote a constant value of that variable or to distinguish between variables. The symbols a_1, a_2, a_3 denote constants. The

syntax $(x_0, y_0), (x_1, y_1)$ denotes coordinates of fixed points; $f(x_1, x_2, x_3, ...x_n)$ denotes a function of *n* variables; ${}_nC_r$ denotes the number of possible combinations of *n* things *r* at a time.

Subset - A set consisting of elements from a given set; it may be the empty set

Example: If *B* = {1,2,3,4,5,6,7} and *A* = {1,2,5}, then *A* is a subset of *B*

Substitution property - Any quantity can be replaced by an equal quantity

Example: If $a + x = b$ and $x = c$ then $a + c = b$

Subtend – To be opposite to, or measure off, as an arc of a circle subtends the central angle of the arc

Subtraction property of equality - If the same or equal quantities are subtracted from same or equal quantities, then the results are equal

Example: If $a = b$ then $a - c = b - c$

Sum and product of roots of a quadratic equation - For a quadratic equation $ax^2 + bx + c = 0, a \neq 0,$ whose roots are x_1 and x_2, the sum of the roots is $x_1 + x_2 = -\frac{b}{a},$ and the product of the roots is $x_1 \times x_2 = \frac{c}{a}$

Sum of a finite geometric series - To determine the sum of the first n terms of a geometric series, $S_n = \frac{a_1(1-r^n)}{1-r}, r \neq 1$, where a_1 is the first term and r is the common ratio

Sum of a finite arithmetic series - To determine the sum of the first n terms of an arithmetic series, $S_n = \frac{n}{2}(a_1 + a_n)$, where a_1 is the first term, and a_n is the nth term

Sum or difference formulas for trigonometric functions - Formulas used to determine trigonometric values of the sum or difference of two angles

$$\sin(A+B) = \sin A \cos B + \cos A \sin B \qquad \sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B \qquad \cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \qquad \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Summation - the act or process of forming a sum

Supplementary angles - Two angles the sum of whose measures is 180 degrees

Surface area - The sum of the areas of all the faces or curved surfaces of a solid figure

Survey - A gathering of facts or opinions by asking people questions through an interview or questionnaire; to query (someone) in order to collect data for the analysis of some aspect of a group or area

Symmetric property of equality - A property of the real numbers that states: If $a = b$ then $b = a$

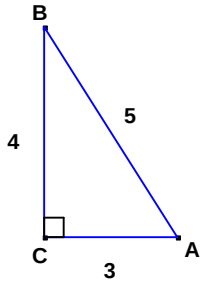
System of equations/inequalities - A set of two or more equations/inequalities; the solution set contains those values that satisfy all of the equations/inequalities in the system

T

Tangent - a straight line or plane that touches a curve or curved surface at a point but does not intersect it at that point

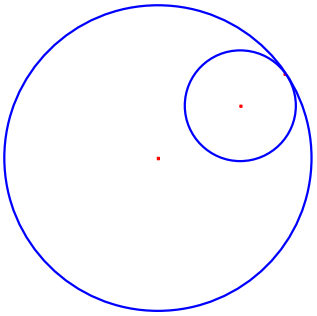
Tangent (of an angle) - For a given acute angle θ in a right triangle, $\tan\theta$ is the ratio of the length of the side opposite the acute angle θ to the length of the side adjacent to the angle θ

Example: In this right triangle, $\tan A = \frac{4}{3}$ and $\tan B = \frac{3}{4}$



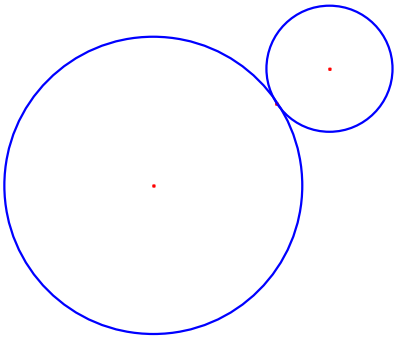
Tangent circles (internal) - Two circles are internally tangent if they intersect in exactly one point and one circle lies in the interior of the other circle

Example:



Tangent circles (external) - Two circles are externally tangent if they meet in exactly one point and neither circle has any points in the interior of the other circle

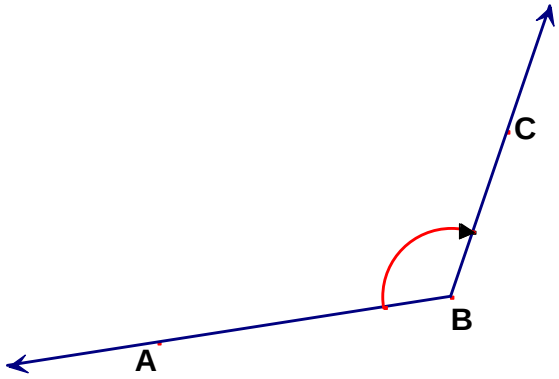
Example:



Tangent line to a circle - A line that intersects a circle in exactly one point

Tangent segment - A line segment that is a subset of a tangent line. This usually refers to the line segment drawn from an external point to the point of tangency.

Terminal side of an angle - If $\angle ABC$ is the directed angle from $\overrightarrow{BA}$ to $\overrightarrow{BC}$ then $\overrightarrow{BC}$ is the terminal side.

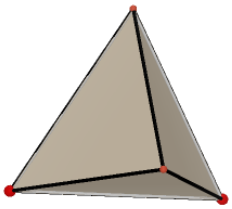


Tessellation - A repeating pattern covering a plane

Examples:

Tetrahedron - A polyhedron with four faces; one of the five Platonic solids that has four equilateral triangles as faces (pyramid)

Example:



Theorem - A general statement that requires proof

Theoretical probability - The chances of events happening as determined by calculating results as they would occur under ideal circumstances

Example: When you roll a die, one of the six possible outcomes is a 2, so the theoretical probability of rolling a 2 is $\frac{1}{6}$; three of the six possible outcomes are odd numbers, so the theoretical probability of rolling an odd number is $\frac{3}{6}$

Three-dimensional space - The set of all points in space. The position of each point can be represented by a unique ordered triple (x,y,z)

Transformation - A one-to-one mapping of points in the plane to points in the plane

Transformational geometry - A method for studying geometry that illustrates congruence and similarity by the use of transformations

Transformational proof - A proof that employs the use of transformations

Transformations of functions and relations - A new function that results from the application of a transformation to a given function

> **Examples:** The function $y=(x-3)^2$ is a translation of the graph of $y=x^2$ three units to the right, while the function $y=(x+3)^2$ is a translation of the graph of $y=x^2$ three units to the left.

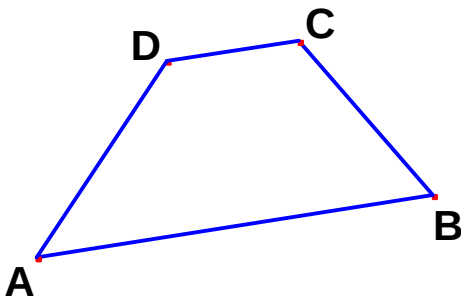
Transitive property of equality - A property of the real numbers that states: If $a=b$ and $b=c$ then $a=c$

Translation - A transformation where every point moves the same direction through the same distance

Transversal - A line that intersects two (or more) other lines in distinct points

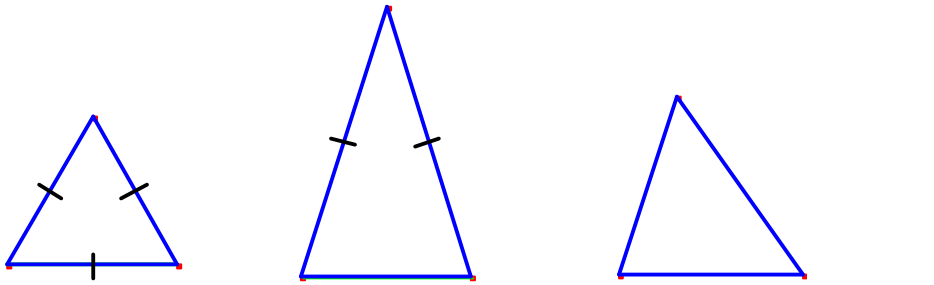
Trapezoid - A quadrilateral with exactly one pair of parallel sides

> **Example:** In the trapezoid below, $\overline{AB}$ ll $\overline{CD}$

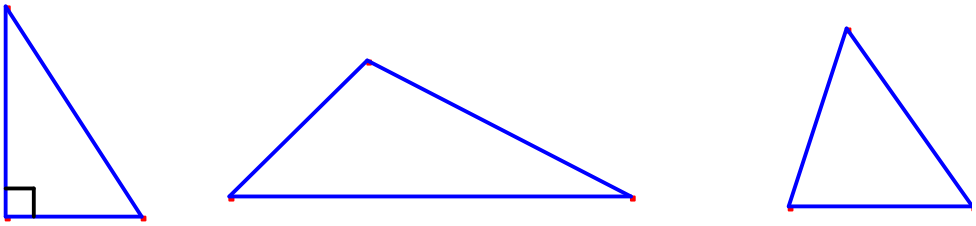


Triangle - A polygon with three sides.

> **Examples:**



equilateral isosceles scalene



right obtuse acute

Triangle inequality theorem - In any triangle, the sum of the lengths of two sides is greater than the length of the third side

Trichotomy property - A property of the real numbers that states: for every x and y, one and only one of the following conditions is true: $x < y$; $x = y$, $x > y$

Trigonometric equation - An equation that uses variables expressed in terms of trigonometric functions

Example: While $x = 3x + 1$ is an algebraic equation, $\sin\theta = 3\sin\theta + 1$ is a trigonometric equation

Trigonometric functions – the ratios between the sides (**A**djacent, **H**ypotenuse, **O**pposite) of a triangle; the ratios are sine, cosine, tangent, cotangent, secant and cosecant; a mnemonic for remembering the trigonometric functions is SOHCAHTOA, where S is $\sin\theta$ or O/H, C is $\cos\theta$ or A/H, and T is $\tan\theta$ or O/A.

$$Sin\theta\,\frac{Opp}{Hyp} \qquad Cos\theta\,\frac{Adj}{Hyp} \qquad Tan\theta\,\frac{Opp}{Adj} \qquad Cot\theta\,\frac{Adj}{Opp} \qquad Sec\theta\,\frac{Hyp}{Adj} \qquad Csc\theta\,\frac{Hyp}{Opp}$$

Trigonometry - the study of the properties of triangles and trigonometric functions and their applications

Trinomial - A polynomial with exactly three terms

Example: $a + 2b + c$, $x^2 - 3x + 5$, $4c^2d + 5cd^2 + 8$

Truth value - A value, (typically T or F), indicating whether a statement is true or false

Two column proof - The outline of a written proof in which the statements and their corresponding reasons are listed in two separate columns

Two-dimensional space - The set of all points in the plane. The position of each point can be represented by a unique ordered pair (x,y). Figures such as angles, pairs of parallel and intersecting lines, circles and polygons exist in two-dimensional space.

U

Undefined - An expression in mathematics which does not have meaning and therefore is not assigned a value

Example: When x=4, the expression $\frac{x+3}{x-4}$ is undefined

When x<2, the expression $\sqrt{x-2}$ is undefined in the real numbers

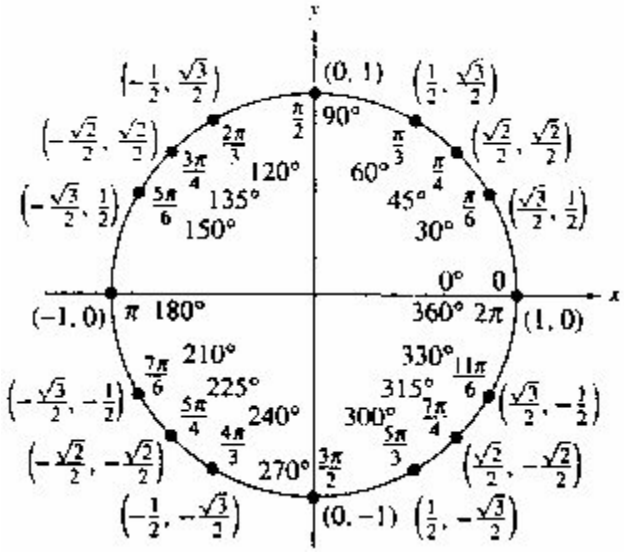
When x<0, the expression $\log_5 x$ is undefined

Undefined terms - The fundamental components of an axiomatic system whose understanding is agreed upon but not formally defined. In geometry undefined terms traditionally include point, line, and plane.

Union of sets - The union of two or more sets is the set of all distinct elements contained in at least one of the sets

Example: if Set *A* = {2,4,6,8,10} and Set *B* = {1,2,3,4,5,6,7,8,9}, then the union of sets *A* and *B*, written as $A \cup B$, is {1,2,3,4,5,6,7,8,9,10}

Unit circle - The circle of radius 1 with center at the origin; in trigonometry, the unit circle is the circle of radius one centered at the origin (0, 0) in the Cartesian coordinate system in the Euclidean plane



Univariate - A set of data involving one variable

Universe - The set of all possible specified elements from which subsets are formed. Also know as the universal set

V

Valid argument - A logical argument supported by known facts or assumed axioms; an argument in which the premise leads to a conclusion

Variable - A quantity that may assume any one of a set of values; a quantity whose value can change or vary; in algebra, letters such as *a*, *b*, and *c*, often represent variables;

Variance - A measure of variability (able or apt to vary: subject to variation or changes)

Variances (population) - A measure of variability given by the average of squared deviations; if data is taken from the entire population, divide by n when averaging the squared deviations:

$$\text{Population variance} = \frac{\sum(x_i - \overline{x})^2}{n}$$

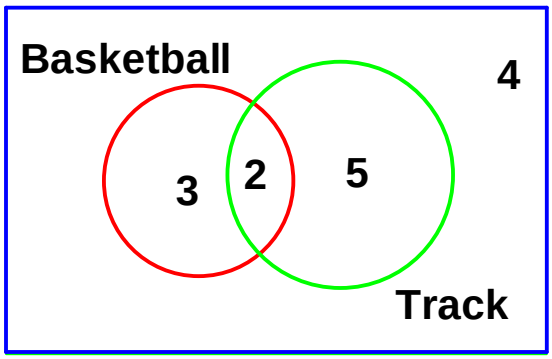
Variance (sample) - A measure of variability given by the average of squared deviations; If data is taken from a sample instead of the entire population, divide by $n-1$ when averaging the squared deviations:

$$\text{Sample variance} = \frac{\sum(x_i - \overline{x})^2}{n-1}$$

Vector - A quantity that has both magnitude and direction; represented geometrically by a directed line segment

Venn diagram - a diagram that uses circles to represent mathematical or logical sets pictorially inside a rectangle (the universal set); elements that are common to more than one set are represented by intersections of the circles

Example: The Venn diagram below shows 14 students. Five students play basketball, seven run track, and two play basketball and run track, three play only basketball, five only run track. Four students do not play basketball or run track.



Vertex of an angle - The point of intersection of the two rays that form the sides of the angle

Vertex of a polygon - A point where the edges of a polygon intersect

Vertex of a cone or pyramid - The fixed point, not in the plane of the base, to which all points on the perimeter of the base are connected

Vertical angles - The two nonadjacent angles formed when two lines intersect

Vertical line test - A vertical line drawn to determine whether or not a relation is a function. A relation is a function if and only if no vertical line intersects the graph of the relation more than once

Volume - A measure of the number of cubic units needed to fill the space inside a solid figure

W

Weighted Average – the average obtained when each observation is multiplied by a factor, called a weight that is an indication of that observation's relative importance.

Whole number – an element of the set of natural numbers, their negatives and zero; a whole number is an integer.

X

***x*-axis** - One of the two intersecting lines used to establish the coordinates of points in the Cartesian plane; in that plane, the line whose equation is $y = 0$; in space the axis perpendicular to the yz-plane

***x*-coordinate** - The first coordinate in any (x,y) ordered pair; the number represents how many units the point is located to the left or right of the *y*-axis; also called abscissa

***x*-intercept** - The point at which the graph of a relation intercepts the *x*-axis; the ordered pair for this point has a value of *(x, 0)*; to find *x*, let $y = 0$ and solve the original equation for *x*

> **Example**: The equation $y = 8 + 2x$ has an *x*-intercept of -4

Y

***y*-axis** - One of the two intersecting lines used to establish the coordinates of points in the Cartesian plane; in that plane, the line whose equation is $x = 0$; in space the axis perpendicular to the xz-plane

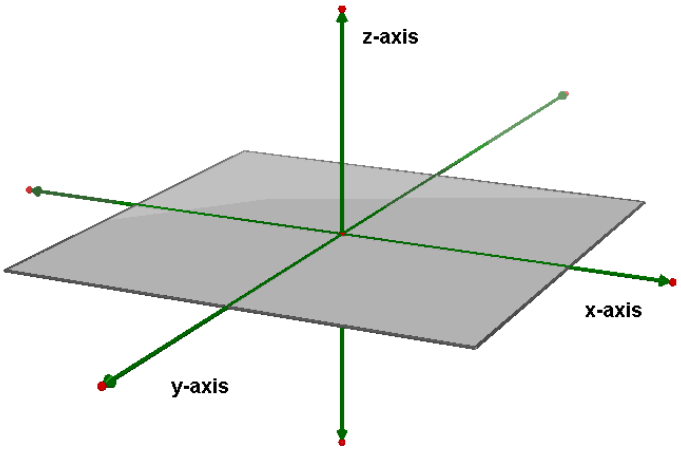
***y*-coordinate** - The second coordinate in any (x,y) ordered pair; the number represents how many units the point is located above or below of the *x*-axis; also called ordinate

***y*-intercept** - The point at which a graph of a relation intercepts the *y*-axis; the ordered pair for this point has a value of *(0, y)*; to find *y*, let $x = 0$ and solve the original equation for *y*

> **Example**: The equation $y = 8 + 2x$ has a y-intercept of 8

Z

z-axis - A line perpendicular to the plane determined by the x-axis and y-axis at their point of intersection; this axis is used as a reference to determine the third component of the ordered triple (x, y, z)



***z*-coordinate** - The third coordinate in any (x,y,z) ordered triple; the number represents how many units the point is located above or below of the *xy*-plane

Zero - As a digit, 0 is used as a placeholder in numeral systems using positional notation; acts as the additive identity of the integers, real numbers, and other algebraic structures

Zero of a function - Any value of the independent variable that makes the value of the function equal to 0

Zero product property - If *a* and *b* are real numbers, then *ab* = 0 if and only if a = *0* or *b* = *0, or a and b = 0*