

Trigonometric Functions and the Unit Circle

Deg	Rad	$\sin\theta \frac{Opp}{Hyp}$	$\cos\theta \frac{Adj}{Hyp}$	$\tan\theta \frac{Opp}{Adj}$	$\cot\theta \frac{Adj}{Opp}$	$\sec\theta \frac{Hyp}{Adj}$	$\csc\theta \frac{Hyp}{Opp}$	Quadrant $r = \sqrt{x^2 + y^2} \neq 0$
		$\sin\theta(y)$ $\frac{b}{c}, \frac{y}{r}$	$\cos\theta(x)$ $\frac{a}{c}, \frac{x}{r}$	$\tan\theta = \frac{\sin\theta(y)}{\cos\theta(x)}$ $\frac{b}{a}, \frac{y}{x}$	$\frac{1}{\tan\theta}$ $\frac{a}{b}, \frac{x}{y}$	$\frac{1}{\cos\theta}$ $\frac{c}{a}, \frac{r}{x}$	$\frac{1}{\sin\theta}$ $\frac{c}{b}, \frac{r}{y}$	
0	0	0	1	0	Undefined	1	Undefined	x-y coordinates (1,0)
30	$\frac{\pi}{6}$ (.542)	$\frac{1}{2}$ (.5)	$\frac{\sqrt{3}}{2}$ (.866)	$\frac{\sqrt{3}}{3}$ (.577)	$\frac{3\sqrt{3}}{3}$	$\frac{2\sqrt{3}}{3}$	2	I - $\sin\theta+$, $\cos\theta+$, $\tan\theta+$, $\cot\theta+$, $\sec\theta+$, $\csc\theta+$
45	$\frac{\pi}{4}$ (.785)	$\frac{\sqrt{2}}{2}$ (.707)	$\frac{\sqrt{2}}{2}$ (.707)	1	1	$\sqrt{2}$	$\sqrt{2}$	I - $\sin\theta+$, $\cos\theta+$, $\tan\theta+$, $\cot\theta+$, $\sec\theta+$, $\csc\theta+$
60	$\frac{\pi}{3}$ (1.04)	$\frac{\sqrt{3}}{2}$ (.866)	$\frac{1}{2}$ (.5)	$\sqrt{3}$ (1.73)	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	I - $\sin\theta+$, $\cos\theta+$, $\tan\theta+$, $\cot\theta+$, $\sec\theta+$, $\csc\theta+$
90	$\frac{\pi}{2}$ (1.57)	1	0	Undefined	0	Undefined	1	x-y coordinates (0,1)
120	$\frac{2\pi}{3}$ (2.1)	$\frac{\sqrt{3}}{2}$ (.866)	$-\frac{1}{2}$ (-.5)	$-\sqrt{3}$ (-1.73)	$-\frac{\sqrt{3}}{3}$	-2	$\frac{2\sqrt{3}}{3}$	II - $\sin\theta+$, $\cos\theta-$, $\tan\theta-$, $\cot\theta-$, $\sec\theta-$, $\csc\theta+$
135	$\frac{3\pi}{4}$ (2.35)	$\frac{\sqrt{2}}{2}$ (.707)	$-\frac{\sqrt{2}}{2}$	-1	-1	$-\sqrt{2}$	$\sqrt{2}$	II - $\sin\theta+$, $\cos\theta-$, $\tan\theta-$, $\cot\theta-$, $\sec\theta-$, $\csc\theta+$
150	$\frac{5\pi}{6}$ (2.61)	$\frac{1}{2}$ (.5)	$-\frac{\sqrt{3}}{2}$	$-\sqrt{3}$ (-1.73)	$-\sqrt{3}$	$-\frac{2\sqrt{3}}{3}$	2	II - $\sin\theta+$, $\cos\theta-$, $\tan\theta-$, $\cot\theta-$, $\sec\theta-$, $\csc\theta+$
180	π (3.14)	0	-1	0	Undefined	-1	Undefined	x-y coordinates (-1,0)

210	$\frac{7\pi}{6}$ (3.66)	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	$-\frac{2\sqrt{3}}{3}$	-2	III - $\text{Sin}\theta-, \text{Cos}\theta-, \text{Tan}\theta+, \text{Cot}\theta+, \text{Sec}\theta-, \text{Csc}\theta-$
225	$\frac{5\pi}{4}$ (3.92)	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	1	1	$-\sqrt{2}$	$-\sqrt{2}$	III - $\text{Sin}\theta-, \text{Cos}\theta-, \text{Tan}\theta+, \text{Cot}\theta+, \text{Sec}\theta-, \text{Csc}\theta-$
240	$\frac{4\pi}{3}$ (4.18)	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$	-2	$-\frac{2\sqrt{3}}{3}$	III - $\text{Sin}\theta-, \text{Cos}\theta-, \text{Tan}\theta+, \text{Cot}\theta+, \text{Sec}\theta-, \text{Csc}\theta-$
270	$\frac{3\pi}{2}$ (4.71)	-1	0	Undefined	0	Undefined	-1	x-y coordinates (0,-1)
300	$\frac{5\pi}{3}$ (5.23)	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{3}$	$-\sqrt{3}$	2	$-\frac{2\sqrt{3}}{3}$	IV - $\text{Sin}\theta-, \text{Cos}\theta+, \text{Tan}\theta-, \text{Cot}\theta-, \text{Sec}\theta+, \text{Csc}\theta-$
315	$\frac{7\pi}{4}$ (5.49)	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	-1	-1	$\sqrt{2}$	$-\sqrt{2}$	IV - $\text{Sin}\theta-, \text{Cos}\theta+, \text{Tan}\theta-, \text{Cot}\theta-, \text{Sec}\theta+, \text{Csc}\theta-$
330	$\frac{11\pi}{6}$ (5.75)	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	$-\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	-2	IV - $\text{Sin}\theta-, \text{Cos}\theta+, \text{Tan}\theta-, \text{Cot}\theta-, \text{Sec}\theta+, \text{Csc}\theta-$
360	2π (6.28)	0	1	0	Undefined	1	Undefined	x-y coordinates (1,0)

Commonly Used Formulas

Radian Measure

The radian measure of a central angle subtending an arc length s in a circle of radius r is $\theta = s/r$. Radian and degree measure are related as follows, where θ_d is the measure of the angle in degrees and θ_r is the measure of the angle in radians.

$$\frac{\theta_d}{180^\circ} = \frac{\theta_r}{\pi}$$

Radians to degrees

$$\theta_d = \frac{\theta_r 180}{\pi}$$

Degrees to radians

$$\theta_r = \frac{\pi \theta_d}{180}$$

$$1 \text{ radian} = \frac{180^\circ}{\pi} = 57^\circ 17' 45''$$

$$1 \text{ degree} = \frac{\pi}{180^\circ} = .01745 \text{ radians}$$

Arc length (s)

Radian Measure

$$s = r\theta$$

Degree Measure

$$s = \frac{\pi}{180} r\theta$$

Proportion relating central angles and arcs

$$\frac{\theta}{360^\circ} = \frac{s}{C}, \text{ where } s \text{ is equal to the arc length and } C \text{ is the circumference } (C = 2\pi r)$$

Linear velocity (V) – arc length per unit time

$$V = \frac{s}{t}$$

Where s is arc length and t is time

Linear velocity is equal to the radius times the angular velocity ω

$$V = r\omega$$

Angular velocity – central angle swept out in radians per unit time

$$\omega = \frac{\theta}{t}$$

Area of a Sector

Radian Measure

$$A = \frac{1}{2}r^2\theta$$

Degree Measure

$$A = \frac{\pi}{360}r^2\theta$$

Where r is the radius