Trigonometric Identities and Formulas

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta}$$
$$\sec\theta = \frac{1}{\cos\theta}$$
$$\cot\theta = \frac{1}{\tan\theta}$$

Quotient Identities

$$\sin\theta = \frac{\tan\theta}{\sec\theta}, \qquad \cos\theta = \frac{\cot\theta}{\csc\theta}, \qquad \tan\theta = \frac{\sin\theta}{\cos\theta}$$
$$\csc\theta = \frac{\sec\theta}{\tan\theta}, \qquad \sec\theta = \frac{\csc\theta}{\cot\theta}, \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1, \qquad \sin^2\theta = 1 - \cos^2\theta, \qquad \cos^2\theta = 1 - \sin^2\theta$$
$$\csc^2\theta - \cot^2\theta = 1, \qquad 1 + \cot^2\theta = \csc^2\theta, \qquad \cot^2\theta = \csc^2\theta - 1$$
$$\sec^2\theta - \tan^2\theta = 1, \qquad 1 + \tan^2\theta = \sec^2\theta, \qquad \tan^2\theta = \sec^2\theta - 1$$

Reduction (Even/Odd) Identities

$$\sin(-\theta) = -\sin\theta, \qquad \sin\theta = -\sin(\theta - \pi)$$
$$\cos(-\theta) = \cos\theta, \qquad \cos\theta = -\cos(\theta - \pi)$$
$$\tan(-\theta) = -\tan\theta, \qquad \tan\theta = \tan(\theta - \pi)$$

Sum or Difference of Two Angles

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$
$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}, \qquad \tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$
$$\cot(\alpha + \beta) = \frac{\cot\beta\cot\alpha - 1}{\cot\beta + \cot\alpha}, \qquad \cot(\alpha - \beta) = \frac{\cot\beta\cot\alpha + 1}{\cot\beta - \cot\alpha}$$

Where α and β are the Greek letters alpha and beta and represent the two angles.

Half Angle Formulas

$$\sin^2\theta = \frac{1-\cos 2\theta}{2} \;=\; \sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2} \;=\; \cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan^2\theta = \frac{1-\cos 2\theta}{1+\cos 2\theta} \;=\; \tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

$$\cot^2\theta = \frac{1+\cos 2\theta}{1-\cos 2\theta} \;=\; \cot\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}$$

Double Angle Formulas

$$\sin 2\theta = 2\sin\theta\cos\theta = \frac{2\tan\theta}{1+\tan^2\theta}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$\cot 2\theta = \frac{\cot^2\theta - 1}{2\cot\theta}$$

Product Relations

$\sin\theta = \tan\theta\cos\theta$, $\quad$ $\cos\theta = \cot\theta\sin\theta$

$\tan\theta = \sin\theta\sec\theta$, $\quad$ $\cot\theta = \cos\theta\csc\theta$

$\sec\theta = \csc\theta\tan\theta$, $\quad$ $\csc\theta = \sec\theta\cot\theta$

Sum/Difference to Product Formulas

$$\sin\alpha+\sin\beta=2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$
$$\sin\alpha-\sin\beta=2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$
$$\cos\alpha+\cos\beta=2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$
$$\cos\alpha-\cos\beta=-2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$
$$\tan\alpha+\tan\beta=\frac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta}$$
$$\tan\alpha-\tan\beta=\frac{\sin(\alpha-\beta)}{\cos\alpha\cos\beta}$$
$$\cot\alpha+\cot\beta=\frac{\sin(\alpha+\beta)}{\sin\alpha\sin\beta}$$
$$\cot\alpha-\cot\beta=\frac{\sin(\beta-a)}{\sin\alpha\sin\beta}$$

Product to Sum/Difference Formulas

$$\sin\alpha\sin\beta=\frac{1}{2}\left[\cos(\alpha-\beta)-\cos(\alpha+\beta)\right]$$
$$\cos\alpha\cos\beta=\frac{1}{2}\left[\cos(\alpha-\beta)+\cos(\alpha+\beta)\right]$$
$$\sin\alpha\cos\beta=\frac{1}{2}\left[\sin(\alpha+\beta)+\sin(\alpha-\beta)\right]$$
$$\cos\alpha\sin\beta=\frac{1}{2}\left[\sin(\alpha+\beta)-\sin(\alpha-\beta)\right]$$

Law of Sines

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos A,\ \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$b^2 = c^2 + a^2 - 2ca\cos B,\ \cos B = \frac{c^2 + a^2 - b^2}{2ca}$$

$$c^2 = a^2 + b^2 - 2ab\cos C,\ \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$